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UNITED STATES DEPARTMENT OF AGRICULTURE
BUREAU OF PUBLIC ROADS

Washington, D. C. August, 1927

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ANALYSIS OF CONCRETE ARCHES [a]

PART I.—DERIVATION OF FORMULAS AND ANALYSIS OF SYMMETRICAL ARCHES

By W. P. LINTON, Highway Bridge Engineer, and C. D. GEISLER, Associate Highway Bridge Engineer, United States Bureau of Public Roads [1]

PRACTICALLY all arches are designed by tentatively proportioning the structure using an empirical formula, or basing the design on the results of previous experience, and then computing the stresses in the tentative design and making such changes as seem to be desirable. This is the procedure followed generally in the design of framed structures, but in the case of arches it is a tedious and complicated process. In 1909 the first-named author attempted to systematize this procedure by developing a standard set of forms for tabulating the computations in such a way that the work is as nearly mechanical as possible and both the labor and the probability of error is greatly reduced. It has been found possible to develop a method whereby much of the work is the same for all arches and the results of this portion of the computations can be placed on the tracings which are used for printing the forms. The method has been in use since 1909, and it has been found to be satisfactory in practice.

No attempt will be made to discuss procedure in determining the curve of the arch ring or its thickness at various points, as the method of calculation presented here may be used in combination with any of the empirical or semirational formulas found in various textbooks. The derivation of the formulas on which the method is based is not greatly different from that found in other places. It is not necessary that these derivations be at hand when using the forms for arch design, but they are included for convenience and completeness. The engineer who is familiar with arch design can proceed at once to the explanation of the forms under the heading, Calculations for a Symmetrical Arch.

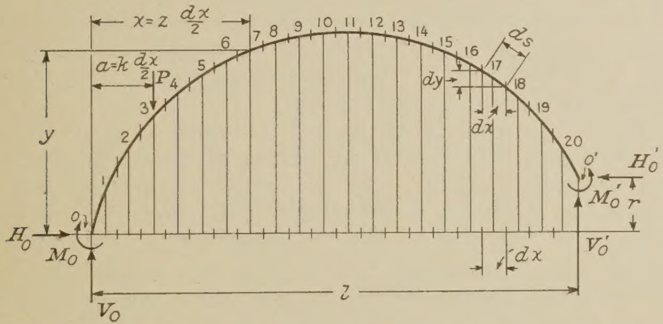


FIG. 1.—SKETCH OF AXIS OF ARCH RING TO SHOW MEANING OF SYMBOLS USED

NOMENCLATURE

The following is the nomenclature used in this discussion. Figures 1, 2, 3, and 4 also illustrate the meaning of many of the symbols, and in each of these figures the origin of coordinates is taken at the left support. All dimensions are in feet unless otherwise stated.

A_c = the area of the concrete in a radial section of the arch ring.

A_s = the area of the steel in a radial section of the arch ring.

$A = A_c + A_s$.

a = the horizontal distance from the origin to the point of application of a concentrated load. $a = k\frac{dx}{2}$ when the span is divided into a number of equal parts, each equal to dx.

$B = \frac{1}{2}\sum_0^l z\Delta\left(y - \frac{\Sigma y\Delta}{\Sigma\Delta}\right)$, a term in the denominator of the equation for V_o for unsymmetrical arches and is independent of the loading.

$C = \frac{1}{dx}\sum_0^l y\Delta\left(y - \frac{\Sigma y\Delta}{\Sigma\Delta}\right) + \sum_0^l \frac{\cos\phi}{A}$, a term in the denominator of certain equations and is independent of the loading.

d' = the distance from the center of the reinforcing steel to the surface of the concrete.

Dl = the change in the span length due to deformation of the arch ring.

Ds = the change in the length of the arch axis due to deformation of the arch ring.

Dr = the change in the relative elevation of the supports due to deformation of the arch ring.

$D(dx)$ = the change in length of dx due to deformation of the arch ring.

$D(ds)$ = the change in length of ds due to deformation of the arch ring.

$D\theta$ = the change in the angle between the end tangents due to deformation of the arch ring.

$D(d\phi)$ = the change in the angle $d\phi$ due to deformation of the arch ring.

$D(dy)$ = the change in length of dy due to deformation of the arch ring.

E = Young's modulus of elasticity.

E_c = Young's modulus of elasticity for concrete.

E_s = Young's modulus of elasticity for steel.

e = the coefficient of expansion due to a change of temperature.

F = a term in the denominator of the equation for V_o.

$F = \frac{1}{2}\sum_0^l z\Delta\left(z - \frac{\Sigma z\Delta}{\Sigma\Delta}\right)$ for unsymmetrical arches and

$F = \frac{1}{2}\sum_0^l z^2\Delta - 200\ \Sigma\Delta$ for symmetrical arches.

f_c = the stress in the concrete.

f_s = the stress in the steel.

$G = \frac{1}{dx}\sum_o^l z\Delta\left(y - \frac{\Sigma y\Delta}{\Sigma\Delta}\right)$, a term in the denominator of the equation for V_o in unsymmetrical arches. G is independent of the loading.

H = the horizontal thrust; that is, the horizontal component of the thrust T.

H_o = the horizontal thrust at the left support.

H_x = the horizontal thrust at a point distant x from the left support.

H_t = the horizontal thrust caused by a change of temperature.

h = the thickness of the arch ring at any point.

I = the moment of inertia of a radial section of the arch ring about the axis of the arch ring.

I_c = the moment of inertia of the concrete in a radial section of the arch ring about the axis of the arch ring.

I_s = the moment of inertia of the reinforcing steel about the axis of the arch ring.

I_o = the moment of inertia of the steel about its own axis.

j = the distance from the axis of the arch to the center of the reinforcement.

$k = \frac{2a}{dx}$ when the span is divided into a number of equal parts each equal to dx.

l = the span of the arch axis.

M = the bending moment.

M_o = the bending moment at the left support.

M_x = the bending moment at a point a distance x from the left support.

M_t = the bending moment caused by a change of temperature.

$m_x = V_o x - \Sigma P(x-a)$, a term which would be the bending moment if the arch were a simple beam on two supports. When P is unity, $m_x = [V_o z - (z-k)]\frac{dx}{2}$.

[a] Reprinted from Public Roads, vol. 8, Nos. 4 and 5, June and July, 1927.

[1] The method of arch analysis described in this article and the one to follow on unsymmetrical arches was developed by W. P. Linton. C. D. Geisler collaborated in preparing the articles for publication.

N = the normal component of the resultant forces acting on a radial section of the arch ring.

$n = \frac{E_s}{E_c} = 15.$

P = a vertical concentrated load.

$p = \frac{N}{A}$, the average stress on a radial section.

r = the difference in elevation of the two supports. $r = 0$ for a symmetrical arch.

s = the length of the arch axis.

ds = the length of one division of the arch axis.

T = the resultant thrust on a radial section of the arch.

t = the number of degrees of rise or fall of temperature.

u = the eccentricity of the normal thrust N.

V = the vertical component of the thrust T at any section.

V_o = the vertical reaction at the left support.

V_x = the vertical component of the thrust T at a point distant x from the left support.

x = the abscissa of any point on the arch axis. $x = z\frac{dx}{2}$.

dx = the horizontal projection of one division of the arch axis.

y = the ordinate of any point on the arch axis.

dy = the vertical projection of one division of the arch axis.

$z = \frac{2x}{dx}$ when the span is divided into a number of equal parts each equal to dx.

$\Delta = \frac{ds}{I}$

θ = the angle between the end tangents to the arch axis (fig. 3).

$\sum\limits_o^a$ = the summation of all terms between the left support and the concentrated load distant a from the left support.

$\sum\limits_a^l$ = the summation of all terms from the concentrated load to the right support.

$\sum\limits_o^l$ = the summation of all terms from one support to the other.

$\Sigma = \sum\limits_o^l$. When no limits are specified, it is understood that the summation is to be taken from one support to the other.

ϕ = the angle which any radial section makes with the vertical. (See fig. 3.)

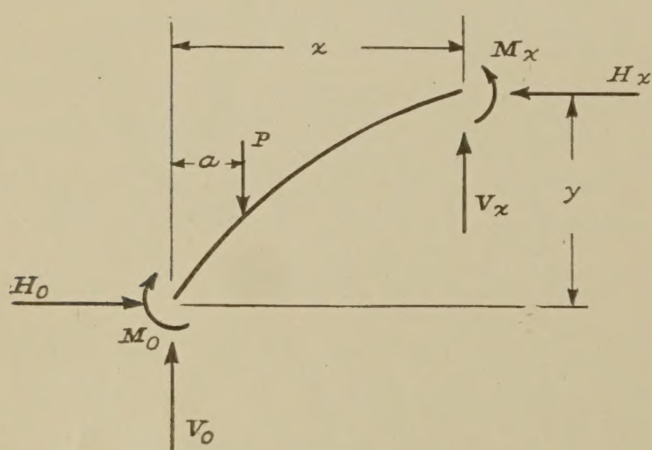


FIG. 2.—HINGELESS ARCH ACTED UPON BY LOAD P

DERIVATION OF FORMULAS

A hingeless arch is statically indeterminate, that is, the stresses in it can not be computed from the principles of statics alone. Let Figure 2 represent a portion of a hingeless arch acted upon by one or more loads P and held in equilibrium by the forces and moments shown. There are six unknown quantities H_o, V_o, M_o, H_x, V_x, and M_x, but only three independent equations can be written from statics. For a single vertical load these equations are as follows:

$$H_x = H_o.$$

$$V_x = V_o - P.$$

$$M_x = M_o + V_o x - H_o y - P(x-a).$$

To determine the six unknown quantities, three additional equations must be supplied from some other source, and they may be derived by considering the elastic properties of the arch ring.

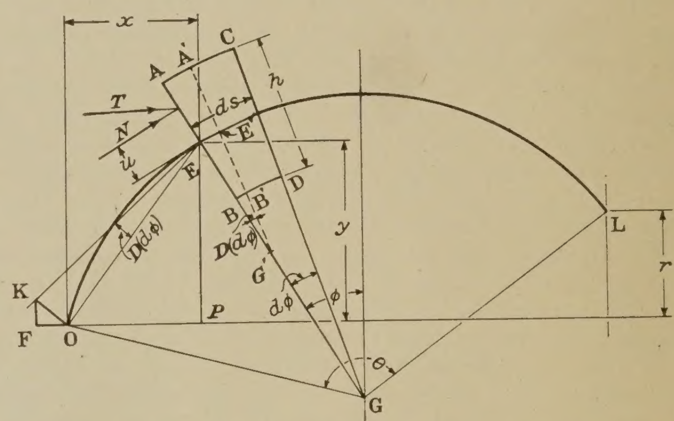


FIG. 3.—DIAGRAM FOR USE IN DERIVATION OF FORMULAS FOR STRESS IN AN ELASTIC ARCH

Referring to Figure 3, let ABCD represent a very small portion of the arch ring, cut out by radial planes AB and CD. Let the length of this small segment, measured along its center line, be ds and let the angle between the planes AB and CD be $d\phi$. Both ds and $d\phi$ are very small, and since the depth AB of the arch ring is small compared with the radius BG, the lengths AC and BD may be considered equal to ds without appreciable error in the results.

When the arch is subjected to loads or a change of temperature, there will be stress and deformation in the element ABCD. If we consider ABCD to be separated from the rest of the arch ring, the internal stresses in it must be held in equilibrium by external forces, so let us assume that T is the resultant of all the external forces acting on the plane AB and N the component of T perpendicular to AB. There is also a component of T parallel to AB, but it will cause only shear in the arch ring, which will be small and may be neglected. Neglecting the component parallel to AB is equivalent to assuming that N is the resultant, which we shall do. The forces acting on the plane AB are of course not concentrated in N, but are distributed over the plane, and we shall assume that they vary uniformly from the value at A to the value at B. Then if A is the area of the plane AB, the unit force or the unit internal stress acting at A will be $\left(\frac{N}{A} + \frac{Mh}{2I}\right)$, and at B, $\left(\frac{N}{A} - \frac{Mh}{2I}\right)$.

M, the bending moment is equal to Nu, u being the distance from the axis of the arch to the line of action of N. M is considered positive when u is positive, that is, when N acts above the axis of the arch, and negative when u is negative.

Consider first the elasticity of only the element ABCD and assume that the arch is fixed at the point L and free to move at point O and that the entire ring, with the exception of the element ABCD, is rigid. The plane CD will then be fixed and the plane AB will move until the internal stresses in the element ABCD are in equilibrium with the bending moment M and the thrust N. The plane AB will move to some position as A′B′. The unit stress in the extreme fiber AC will be $\left(\frac{N}{A}+\frac{Mh}{2I}\right)$, and since we can assume the length of this extreme fiber to be equal to the average length ds of the element, the distance AA′ will be $-\left(\frac{N}{A}+\frac{Mh}{2I}\right)\frac{ds}{E}$. Likewise the unit stress in the fiber BD will be $\left(\frac{N}{A}-\frac{Mh}{2I}\right)$, and the distance BB′ will be $-\left(\frac{N}{A}-\frac{Mh}{2I}\right)\frac{ds}{E}$, in which E is the modulus of elasticity of the material comprising the arch ring. AA′ and BB′ are both negative quantities because they represent decreases in the lengths AC and BD, respectively, and as it is assumed that N is positive, minus signs must be placed in front of the two quantities, as shown.

The movement of the plane AB may be considered as taking place in two separate stages: First, as moving along the axis of the arch ring from E to E′, during which it remains parallel to its original position; and second, as rotating around the point E′, through an angle $D(d\phi)$ and taking the position A′B′. The distance EE′ which the plane AB moves along the axis of the arch ring is called $D(ds)$ because that is the change in the length of ds. Likewise, the angle through which it turns we call $D(d\phi)$ because that is the change in the angle $d\phi$.

When ds and $d\phi$ increase in magnitude, $D(ds)$ and $D(d\phi)$ will be considered as positive, and when ds and $d\phi$ decrease in magnitude $D(ds)$ and $D(d\phi)$ will be considered as negative. In the figure, the angle $d\phi$ is decreased by the deformation; therefore the angle $D(d\phi)$ is negative.

The angle $D(d\phi)$ is equal to $\frac{\text{AA}'-\text{BB}'}{h}$; and since AA′ is equal to $-\left(\frac{N}{A}+\frac{Mh}{2I}\right)\frac{ds}{E}$, and BB′ is equal to $-\left(\frac{N}{A}-\frac{Mh}{2I}\right)\frac{ds}{E}$, the angle $D(d\phi)$ is equal to

$$\left[-\left(\frac{N}{A}+\frac{Mh}{2I}\right)\frac{ds}{E}+\left(\frac{N}{A}-\frac{Mh}{2I}\right)\frac{ds}{E}\right]\frac{1}{h},$$

which reduces to $-\frac{Mds}{EI}$.

$$\text{Therefore } D(d\phi)=-M\frac{ds}{EI}\text{ - }(1)$$

Now consider the effect on the point O of the rotating of the plane AB through the angle $D(d\phi)$. The arch ring is free to move at O. Since that part of the ring between O and E is rigid, when the plane AB turns through the angle $D(d\phi)$ the line EO will likewise describe the same angle. During this rotation the point O will move through an arc of a circle to K. OK will then be equal to OE times the angle $D(d\phi)$; and since the angle $D(d\phi)$ is negative, OK $=-$OE $D(d\phi)$. Since the angle $D(d\phi)$ is very small, we may consider that OK is a straight line and that the angle EOK is a right angle. From geometry, the angle FOK is equal to the angle PEO.

Then, $\frac{\text{OF}}{\text{OK}}=\frac{\text{EP}}{\text{OE}}=\frac{y}{\text{OE}}$. (OF represents an increase in span of arch and is therefore positive.)

$$\text{OF}=\text{OK}\frac{y}{\text{OE}}=-\text{OE}\,D(d\phi)\frac{y}{\text{OE}}$$

$$\text{Or, OF}=-D(d\phi)y$$

$$\text{But, } D(d\phi)=-M\frac{ds}{EI}$$

$$\text{Therefore, OF}=+My\frac{ds}{EI}\text{ - - - - - - - - - - }(2)$$

Also, $\frac{\text{FK}}{\text{OK}}=\frac{\text{OP}}{\text{OE}}=\frac{x}{\text{OE}}$. (FK represents an upward movement of the arch at point O and is therefore positive.)

$$\text{FK}=\text{OK}\frac{x}{\text{OE}}=-\text{OE}\,D(d\phi)\frac{x}{\text{OE}}$$

$$\text{Or, FK}=-D(d\phi)\,x$$

$$\text{By substitution, FK}=+Mx\frac{ds}{EI}\text{ - - - - - - - - - - - - - - - - - - }(3)$$

Now let us return and consider the effect on the point O of the other stage of the movement of the plane AB, that is, its movement along the axis of the arch during which it remains parallel to its original position. It is apparent that during this movement of the plane AB the change in position of the point O will correspond exactly with that of the point E; that is, point O will move a distance equal to $D(ds)$. Now $D(ds)$ is equal to $\frac{1}{2}$(AA′ + BB′), or to

$$\frac{1}{2}\left[-\left(\frac{N}{A}+\frac{Mh}{2I}\right)\frac{ds}{E}-\left(\frac{N}{A}-\frac{Mh}{2I}\right)\frac{ds}{E}\right],$$

which reduces to $-\frac{Nds}{AE}$.

$$\text{Therefore, } D(ds)=-\frac{Nds}{AE}\text{ - }(4)$$

dx and dy are the horizontal and vertical projections of ds, and since ϕ is equal to the angle which ds makes with the horizontal, $dx=ds\cos\phi$, and $dy=ds\sin\phi$; also $D(dx)=D(ds)\cos\phi$ and $D(dy)=D(ds)\sin\phi$.

$$\text{By substitution, } D(dx)=-\frac{Nds}{AE}\cos\phi$$

$$\text{and } D(dy)=-\frac{Nds}{AE}\sin\phi$$

Substituting the values of $\cos\phi=\frac{dx}{ds}$, and $\sin\phi=\frac{dy}{ds}$,

$$D(dx)=-\frac{Ndx}{AE}\text{ - }(5)$$

$$\text{and } D(dy)=-\frac{Ndy}{AE}\text{ - }(6)$$

Equations 5 and 6 give the horizontal and vertical changes in position of the point O caused by the thrust N.

We now have the changes in position of the point O caused by both the bending moment M and the thrust N acting on the element ABCD when the point O is

free to move and all of the arch is rigid except the element ABCD. So far we have not considered what caused the bending moment, M, and the thrust, N, but in the preceding discussion it does not matter. Let us consider, however, that the moment and thrust were produced by vertical loads. Then if the element ABCD undergoes a change of temperature there will be an additional movement of the point O. Since O and the plane, AB, are free to move, this movement will be simply a change of length of ds, which we may call $D_t(ds)$, and is equal to $et(ds)$, in which t is the number of degrees of change of temperature and e is the coefficient of expansion of the material comprising the arch ring. Thus $D_t(ds)=et(ds)$. In the same manner we find that:

$$D_t(dx)=et(dx) \quad (7)$$

$$\text{and } D_t(dy)=et(dy) \quad (8)$$

We now have in equations 1 to 8, inclusive, expressions for the change in the angle $d\phi$ and the horizontal and vertical displacements of point O with respect to point L as produced by the action of both vertical loads and temperature and when only the element ABCD is affected.

Let $D(dl)$ equal the horizontal movement of the point O with respect to L, and let an increase in span length be considered as positive.

Let $D(dr)$ equal the change in elevation of the point O with respect to the point L, and let an increase in elevation of point O be considered as positive.

$$\text{Then, } D(dl)=\text{OF}+D(dx)+D_t(dx)$$

$$D(dr)=\text{FK}-D(dy)-D_t(dy)$$

$$\text{Or, } D(dl)=+My\frac{ds}{EI}-\frac{Ndx}{AE}+et(dx) \quad (9)$$

$$D(dr)=+Mx\frac{ds}{EI}+\frac{Ndy}{AE}-et(dy) \quad (10)$$

In equations 1, 9, and 10 we have expressions for the change in the angle $d\phi$ and the horizontal and vertical displacement of point O relative to point L as produced by the deformation of the element ABCD. If the entire arch ring were elastic, we would find the total displacement of point O relative to point L, as caused by the deformation of the entire arch ring, by taking the sum of the movements of point O as caused by the deformation of all of the elements. Therefore, let

$D\theta$ = the change in the angle θ caused by the deformation of the entire arch ring,

Dl = the change in span length caused by the deformation of the entire arch ring, and

Dr = the change in the elevation of point O relative to point L as caused by the deformation of the entire arch ring.

$$\text{Then, } D\theta=-\frac{1}{E}\sum_o^l M\frac{ds}{I} \quad ^3\ (11)$$

$$Dl=\frac{1}{E}\sum_o^l My\frac{ds}{I}-\frac{1}{E}\sum_o^l\frac{N}{A}dx+etl \quad (12)$$

$$Dr=\frac{1}{E}\sum_o^l Mx\frac{ds}{I}+\frac{1}{E}\sum_o^l\frac{N}{A}dy-etr \quad (13)$$

in which E is a constant and may be taken outside of the summation sign. $\sum_o^l dx$ is equal to l, and $\sum_o^l dy$ is equal to r.

In the previous discussion it has been assumed that the arch ring is fixed at the right end and free to move at the left end, and we have derived equations 11, 12, and 13, which give the amount of the change in the inclination of the tangent to the arch axis at the free end and also the horizontal and vertical movements at that end.

Let us now imagine a horizontal thrust, H_o, a vertical reaction, V_o, and a bending moment, M_o, all applied at the free end and of exactly sufficient intensity to bring the free end of the arch ring back to its original position. Under these conditions $D\theta$, Dl, and Dr will each be equal to zero, so we may write:

$$\sum_o^l M\Delta=0 \quad (14)$$

$$\sum_o^l My\Delta-\sum_o^l\frac{N}{A}dx+etlE=0 \quad (15)$$

$$\sum_o^l Mx\Delta+\sum_o^l\frac{N}{A}dy-etrE=0 \quad (16)$$

in which Δ is equal to $\frac{ds}{I}$.

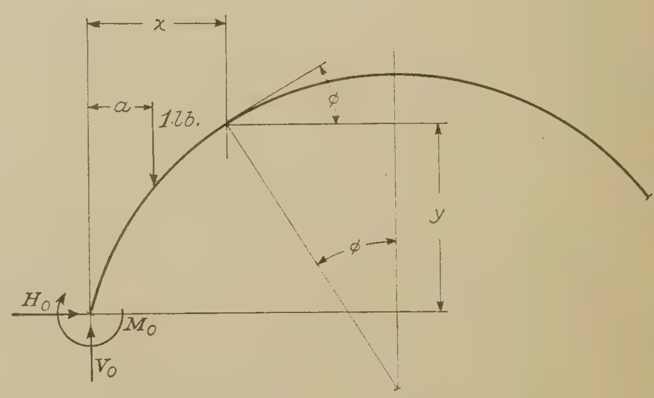


Fig. 4.—Arch Ring With Unit Load at Point Distant a from Support

In these equations, M and N are the bending moment and thrust at any point in the arch ring, of which point x and y are the coordinates. These equations are true for any condition of loading, but for our present purpose we shall consider only a single load of unity placed a distance, a, from the left support (fig. 4).

[3] Throughout this article where limits are given with the summation sign such as $\sum_o^l$ it is intended to indicate that these limits are values of x or in this instance $\sum_{x=o}^{x=l}$. Where no limits are given, they should be assumed as being between $x=l$ and $x=o$.

Then:

$$\left.\begin{aligned} M &= M_o + V_o x - H_o y, && \text{when } x<a \\ M &= M_o + V_o x - H_o y - (x-a), && \text{when } x>a \end{aligned}\right\} \text{-- (17)}$$

$$N = H\cos\phi + V\sin\phi \text{ ------------------- (18)}$$

in which, M_o, V_o, and H_o are, respectively, the bending moment, the vertical reaction, and the horizontal thrust at the left support.

In the following mathematical transformations, x and y are the variables, while a remains constant. Therefore, M_o, V_o, and H_o are constants because they depend on a and not on x or y. For that reason they may be taken outside of the summation signs.

Substituting equations 17 and 18 in equations 14, 15, and 16 we have:

$$\sum_o^a [M_o + V_o x - H_o y]\Delta + \sum_a^l [M_o + V_o x - H_o y - (x-a)]\Delta = 0$$

$$\sum_o^a [M_o + V_o x - H_o y]y\Delta + \sum_a^l [M_o + V_o x - H_o y - (x-a)]y\Delta - \sum_o^l \left(\frac{H\cos\phi + V\sin\phi}{A}\right)dx + etlE = 0$$

$$\sum_o^a [M_o + V_o x - H_o y]x\Delta + \sum_a^l [M_o + V_o x - H_o y - (x-a)]x\Delta + \sum_o^l \left(\frac{H\cos\phi + V\sin\phi}{A}\right)dy - etrE = 0$$

These three equations may be written:

$$M_o\sum_o^l \Delta + V_o\sum_o^l x\Delta - H_o\sum_o^l y\Delta - \sum_a^l (x-a)\Delta = 0$$

$$M_o\sum_o^l y\Delta + V_o\sum_o^l xy\Delta - H_o\sum_o^l y^2\Delta - \sum_a^l (x-a)y\Delta - \sum_o^l H\frac{\cos\phi}{A}dx - \sum_o^l V\frac{\sin\phi}{A}dx + etlE = 0$$

$$M_o\sum_o^l x\Delta + V_o\sum_o^l x^2\Delta - H_o\sum_o^l xy\Delta - \sum_a^l (x-a)x\Delta + \sum_o^l H\frac{\cos\phi}{A}dy + \sum_o^l V\frac{\sin\phi}{A}dy - etrE = 0$$

In the last two equations H and V are still inside the summation sign, but $H = H_o$ and is therefore constant.

Also, $V = V_o$, for $x<a$
$V = V_o - 1$, for $x>a$

Making this substitution in the three equations, we have:

$$M_o\sum_o^l \Delta + V_o\sum_o^l x\Delta - H_o\sum_o^l y\Delta - \sum_a^l (x-a)\Delta = 0$$

$$M_o\sum_o^l y\Delta + V_o\sum_o^l xy\Delta - H_o\sum_o^l y^2\Delta - \sum_a^l (x-a)y\Delta - H_o\sum_o^l \frac{\cos\phi}{A}dx - V_o\sum_o^a \frac{\sin\phi}{A}dx - (V_o - 1)\sum_a^l \frac{\sin\phi}{A}dx + etlE = 0$$

$$M_o\sum_o^l x\Delta + V_o\sum_o^l x^2\Delta - H_o\sum_o^l xy\Delta - \sum_a^l (x-a)x\Delta + H_o\sum_o^l \frac{\cos\phi}{A}dy + V_o\sum_o^a \frac{\sin\phi}{A}dy + (V_o - 1)\sum_a^l \frac{\sin\phi}{A}dy - etrE = 0$$

These three equations may be written:

$$M_o\sum_o^l \Delta + V_o\sum_o^l x\Delta - H_o\sum_o^l y\Delta - \sum_a^l (x-a)\Delta = 0 \text{ ------ (19)}$$

$$M_o\sum_o^l y\Delta + V_o\sum_o^l xy\Delta - H_o\sum_o^l y^2\Delta - \sum_a^l (x-a)y\Delta - H_o\sum_o^l \frac{\cos\phi}{A}dx - V_o\sum_o^l \frac{\sin\phi}{A}dx + \sum_a^l \frac{\sin\phi}{A}dx + etlE = 0 \text{ ---------- (20)}$$

$$M_o\sum_o^l x\Delta + V_o\sum_o^l x^2\Delta - H_o\sum_o^l xy\Delta - \sum_a^l (x-a)x\Delta + H_o\sum_o^l \frac{\cos\phi}{A}dy + V_o\sum_o^l \frac{\sin\phi}{A}dy - \sum_a^l \frac{\sin\phi}{A}dy - etrE = 0 \text{ ---------- (21)}$$

From equation 19

$$M_o = H_o\frac{\Sigma y\Delta}{\Sigma\Delta} - V_o\frac{\Sigma x\Delta}{\Sigma\Delta} + \frac{1}{\Sigma\Delta}\sum_a^l (x-a)\Delta$$

Substituting this value of M_o in equations 20 and 21 and collecting the terms containing H_o and V_o, we have:

$$V_o\left[-\Sigma y\Delta\frac{\Sigma x\Delta}{\Sigma\Delta} + \Sigma xy\Delta - \Sigma\frac{\sin\phi}{A}dx\right] + H_o\left[\Sigma y\Delta\frac{\Sigma y\Delta}{\Sigma\Delta} - \Sigma y^2\Delta - \Sigma\frac{\cos\phi}{A}dx\right]$$

$$= -\frac{\Sigma y\Delta}{\Sigma\Delta}\sum_a^l (x-a)\Delta + \sum_a^l (x-a)y\Delta - \sum_a^l \frac{\sin\phi}{A}dx - etlE, \text{ and}$$

$$V_o\left[-\frac{(\Sigma x\Delta)^2}{\Sigma\Delta}+\Sigma x^2\Delta+\Sigma\frac{\sin\phi}{A}dy\right]$$

$$+H_o\left[\Sigma x\Delta\frac{\Sigma y\Delta}{\Sigma\Delta}-\Sigma xy\Delta+\Sigma\frac{\cos\phi}{A}dy\right]$$

$$=-\frac{\Sigma x\Delta}{\Sigma\Delta}\overset{l}{\Sigma}(x-a)\Delta+\overset{l}{\underset{a}{\Sigma}}(x-a)x\Delta$$

$$+\overset{l}{\underset{a}{\Sigma}}\frac{\sin\phi}{A}dy+etrE=0$$

These two equations may be written:

$$V_o\left[\Sigma x\Delta\left(y-\frac{\Sigma y\Delta}{\Sigma\Delta}\right)-\Sigma\frac{\sin\phi}{A}dx\right]$$

$$-H_o\left[\Sigma y\Delta\left(y-\frac{\Sigma y\Delta}{\Sigma\Delta}\right)+\Sigma\frac{\cos\phi}{A}dx\right]$$

$$=\overset{l}{\underset{a}{\Sigma}}(x-a)\Delta\left(y-\frac{\Sigma y\Delta}{\Sigma\Delta}\right)-\overset{l}{\underset{a}{\Sigma}}\frac{\sin\phi}{A}dx-etlE \qquad (22)$$

$$V_o\left[\Sigma x\Delta\left(x-\frac{\Sigma x\Delta}{\Sigma\Delta}\right)+\Sigma\frac{\sin\phi}{A}dy\right]$$

$$-H_o\left[\Sigma x\Delta\left(y-\frac{\Sigma y\Delta}{\Sigma\Delta}\right)-\Sigma\frac{\cos\phi}{A}dy\right]$$

$$=\overset{l}{\underset{a}{\Sigma}}(x-a)\Delta\left(x-\frac{\Sigma x\Delta}{\Sigma\Delta}\right)+\overset{l}{\underset{a}{\Sigma}}\frac{\sin\phi}{A}dy+etrE \qquad (23)$$

Let $x=z\frac{dx}{2}$ and $a=k\frac{dx}{2}$.

When the span is divided into 20 equal parts each of length dx, $l=40\frac{dx}{2}$. We can consider dx as a constant and z as a variable.

Making these substitutions, and dividing equation 22 by dx and equation 23 by $\frac{(dx)^2}{2}$ we have:

$$V_o\left[\frac{1}{2}\Sigma z\Delta\left(y-\frac{\Sigma y\Delta}{\Sigma\Delta}\right)-\Sigma\frac{\sin\phi}{A}\right]$$

$$-H_o\left[\frac{1}{dx}\Sigma y\Delta\left(y-\frac{\Sigma y\Delta}{\Sigma\Delta}\right)+\Sigma\frac{\cos\phi}{A}\right]$$

$$=\frac{1}{2}\overset{l}{\underset{a}{\Sigma}}(z-k)\Delta\left(y-\frac{\Sigma y\Delta}{\Sigma\Delta}\right)-\overset{l}{\underset{a}{\Sigma}}\frac{\sin\phi}{A}-20\,etE \qquad (24)$$

$$V_o\left[\frac{1}{2}\Sigma z\Delta\left(z-\frac{\Sigma z\Delta}{\Sigma\Delta}\right)+\frac{2}{(dx)^2}\Sigma\frac{\sin\phi}{A}dy\right]$$

$$-H_o\left[\frac{1}{dx}\Sigma z\Delta\left(y-\frac{\Sigma y\Delta}{\Sigma\Delta}\right)-\frac{2}{(dx)^2}\Sigma\frac{\cos\phi}{A}dy\right]$$

$$=\frac{1}{2}\overset{l}{\underset{a}{\Sigma}}(z-k)\Delta\left(z-\frac{\Sigma z\Delta}{\Sigma\Delta}\right)+\frac{2}{(dx)^2}\overset{l}{\underset{a}{\Sigma}}\frac{\sin\phi}{A}dy$$

$$+\frac{2}{(dx)^2}etrE \qquad (25)$$

In the above equations the terms containing $\sin\phi$ and $\cos\phi$ are those which include the effect of the axial or direct stress. To neglect them would be to neglect the axial stress which is sometimes done because it is usually a small part of the total stress. It may not be desirable to neglect this stress entirely, but we can shorten the work considerably by neglecting some of the terms.

The terms $\Sigma\frac{\sin\phi}{A}$, $\frac{2}{(dx)^2}\Sigma\frac{\sin\phi}{A}dy$, $\frac{2}{(dx)^2}\Sigma\frac{\cos\phi}{A}dy$, $\overset{l}{\underset{a}{\Sigma}}\frac{\sin\phi}{A}$, and $\frac{2}{(dx)^2}\overset{l}{\underset{a}{\Sigma}}\frac{\sin\phi}{A}dy$ are all very small compared with the terms to which they are added, and probably no accuracy will be sacrificed by neglecting them. The data upon which all of the terms depend are obtained by scaling a large-scale drawing, and this drawing, while probably more accurate in dimensions than the dimensions of the arch as built is not sufficiently accurate to make the above-mentioned terms significant.

Omitting the above-mentioned terms, we have:

$$V_o\frac{1}{2}\Sigma z\Delta\left(y-\frac{\Sigma y\Delta}{\Sigma\Delta}\right)-H_o\left[\frac{1}{dx}\Sigma y\Delta\left(y-\frac{\Sigma y\Delta}{\Sigma\Delta}\right)+\Sigma\frac{\cos\phi}{A}\right]$$

$$=\frac{1}{2}\overset{l}{\Sigma}(z-k)\Delta\left(y-\frac{\Sigma y\Delta}{\Sigma\Delta}\right)-20\,etE \qquad (26)$$

$$V_o\frac{1}{2}\Sigma z\Delta\left(z-\frac{\Sigma z\Delta}{\Sigma\Delta}\right)-H_o\frac{1}{dx}\Sigma z\Delta\left(y-\frac{\Sigma y\Delta}{\Sigma\Delta}\right)$$

$$=\frac{1}{2}\overset{l}{\underset{a}{\Sigma}}(z-k)\Delta\left(z-\frac{\Sigma z\Delta}{\Sigma\Delta}\right)+\frac{2}{(dx)^2}etr\,E \qquad (27)$$

In these two equations the coefficients of H_o and V_o contain no summations except those for the entire arch ring. For that reason these coefficients are independent of the loading and may be considered as constants.

Therefore we may let:

$$\boldsymbol{B}=\frac{1}{2}\Sigma z\Delta\left(y-\frac{\Sigma y\Delta}{\Sigma\Delta}\right) \qquad (28)$$

$$\boldsymbol{C}=\frac{1}{dx}\Sigma y\Delta\left(y-\frac{\Sigma y\Delta}{\Sigma\Delta}\right)+\Sigma\frac{\cos\phi}{A} \qquad (29)$$

$$\boldsymbol{F}=\frac{1}{2}\Sigma z\Delta\left(z-\frac{\Sigma z\Delta}{\Sigma\Delta}\right) \qquad (30)$$

$$\boldsymbol{G}=\frac{1}{dx}\Sigma z\Delta\left(y-\frac{\Sigma y\Delta}{\Sigma\Delta}\right) \qquad (31)$$

Then the two equations may be written:

$$V_o\boldsymbol{B}-H_o\boldsymbol{C}=\frac{1}{2}\overset{l}{\underset{a}{\Sigma}}(z-k)\Delta\left(y-\frac{\Sigma y\Delta}{\Sigma\Delta}\right)-20\,etE$$

$$V_o\boldsymbol{F}-H_o\boldsymbol{G}=\frac{1}{2}\overset{l}{\underset{a}{\Sigma}}(z-k)\Delta\left(z-\frac{\Sigma z\Delta}{\Sigma\Delta}\right)+\frac{2}{(dx)^2}etrE$$

From the first of these two equations:

$$H_o=V_o\frac{\boldsymbol{B}}{\boldsymbol{C}}-\frac{1}{\boldsymbol{C}}\left[\frac{1}{2}\overset{l}{\underset{a}{\Sigma}}(z-k)\Delta\left(y-\frac{\Sigma y\Delta}{\Sigma\Delta}\right)-20\,etE\right] \qquad (32)$$

and by substitution:

$$V_o=\frac{\frac{1}{2}\sum_a^l(z-k)\Delta\left(z-\frac{\Sigma z\Delta}{\Sigma\Delta}\right)+\frac{2}{(dx)^2}etrE}{F-\frac{BG}{C}}$$

$$-\frac{\frac{G}{C}\left[\frac{1}{2}\sum_a^l(z-k)\Delta\left(y-\frac{\Sigma y\Delta}{\Sigma\Delta}\right)-20\ etE\right]}{F-\frac{BG}{C}}\text{ - - (33)}$$

It is more convenient to consider the effect of the loads and temperature separately. When the effect of the loads is considered without a change of temperature, t in the formulas becomes zero. Then the effect of a change of temperature may be computed by making the terms which contain the effect of the loads equal to zero.

Making this separation and collecting the formulas which are used in computing the stresses in an arch ring, we have:

$$V_o=\frac{\frac{1}{2}\sum_a^l(z-k)\ \Delta\left(z-\frac{\Sigma z\Delta}{\Sigma\Delta}\right)}{F-\frac{BG}{C}}$$

$$-\frac{\frac{G}{C}\frac{1}{2}\sum_a^l(z-k)\ \Delta\left(y-\frac{\Sigma y\Delta}{\Sigma\Delta}\right)}{F-\frac{BG}{C}}\text{ - - - - - - (34)}$$

$$H_o=\frac{V_oB-\frac{1}{2}\sum_a^l(z-k)\ \Delta\left(y-\frac{\Sigma y\Delta}{\Sigma\Delta}\right)}{C}\text{ - - - - - - - - - (35)}$$

$$M_o=\frac{dx}{\Sigma\Delta}\frac{1}{2}\sum_a^l(z-k)\ \Delta+H_o\frac{\Sigma y\Delta}{\Sigma\Delta}-V_o\frac{dx}{2}\frac{\Sigma z\Delta}{\Sigma\Delta}\text{ - - (36)}$$

$$V_x=\begin{cases}V_o,\text{ for }x<a\\ V_o-1,\text{ for }x>a\end{cases}\text{ - (37)}$$

$$H_x=H_o\text{ - (38)}$$

$$M_x=\begin{cases}M_o+V_oz\frac{dx}{2}-H_oy,\text{ for }x<a\\ M_o+[V_oz-(z-k)]\frac{dx}{2}-H_oy,\text{ for }x>a\end{cases}\text{ - - (39)}$$

$$N_x=H_o\cos\phi+V_x\sin\phi\text{ - - - - - - - - - - - - - - - - - - (40)}$$

$$V_t=\frac{\frac{2r}{(dx)^2}+20\frac{G}{C}}{F-\frac{BG}{C}}etE\text{ - (41)}$$

$$H_t=\frac{V_tB+20\ etE}{C}\text{ - (42)}$$

$$M_t=-H_t\left(y-\frac{\Sigma y\Delta}{\Sigma\Delta}\right)+V_t\frac{dx}{2}\left(z-\frac{\Sigma z\Delta}{\Sigma\Delta}\right)\text{ - - - - - - (43)}$$

$$f_c=\frac{N}{A}\pm M\frac{h}{2I}\text{ - (44)}$$

Equations 34 to 44, inclusive, are applicable to all concrete arches with fixed ends, whether symmetrical or unsymmetrical. However, if the arch is symmetrical, some of the terms in the above equations will become zero and the equations will be simplified.

If the arch is symmetrical, the values of Δ will be symmetrical—that is, the value of Δ for point 1′ will be the same as for point 1 and for point 2′ the same as for point 2, etc. The value of z for points 1 and 1′ combined is $(1+39)=40$ and for points 2 and 2′ combined it is $(3+37)=40$ and in the same way for all pairs of symmetrical points combined the value of z is always 40. Therefore, for symmetrical arches $\Sigma z\Delta=20\,\Sigma\Delta$. The values of y are also symmetrical and we have $\Sigma zy\Delta=20\ \Sigma y\Delta$.

These values transform equation 28 as follows:

$$\begin{aligned}B&=\frac{1}{2}\Sigma z\Delta\left(y-\frac{\Sigma y\Delta}{\Sigma\Delta}\right)=\frac{1}{2}\Sigma zy\Delta-\frac{1}{2}\Sigma z\Delta\frac{\Sigma y\Delta}{\Sigma\Delta}\\&=\frac{1}{2}\times20\Sigma y\Delta-\frac{1}{2}\times20\Sigma\Delta\frac{\Sigma y\Delta}{\Sigma\Delta}\\&=10\Sigma y\Delta-10\Sigma y\Delta=0\end{aligned}$$

Therefore, for symmetrical arches

$$B=0$$

$$\begin{aligned}F&=\frac{1}{2}\Sigma z\Delta\left(z-\frac{\Sigma z\Delta}{\Sigma\Delta}\right)=\frac{1}{2}\Sigma z^2\Delta-\frac{1}{2}\Sigma z\Delta\frac{\Sigma z\Delta}{\Sigma\Delta}\\&=\frac{1}{2}\Sigma z^2\Delta-\frac{1}{2}\times20\Sigma\Delta\frac{20\Sigma\Delta}{\Sigma\Delta}\\&=\frac{1}{2}\Sigma z^2\Delta-200\Sigma\Delta\end{aligned}$$

$G=0$ for the same reason as given for B

Therefore, for symmetrical arches we have the following formulas:

FORMULAS FOR SYMMETRICAL ARCHES

$$C=\frac{1}{dx}\Sigma y\Delta\left(y-\frac{\Sigma y\Delta}{\Sigma\Delta}\right)+\Sigma\frac{\cos\phi}{A}$$

$$F=\frac{1}{2}\Sigma z^2\Delta-200\Sigma\Delta$$

$$V_o=\frac{\frac{1}{2}\sum_a^l(z-k)\ (z-20)\ \Delta}{F}\text{ - (45)}$$

$$H_o=\frac{-\frac{1}{2}\sum_a^l(z-k)\Delta\left(y-\frac{\Sigma y\Delta}{\Sigma\Delta}\right)}{C}\text{ - - - - - - - - - - - - - - - - (46)}$$

$$M_o=\frac{dx}{\Sigma\Delta}\frac{1}{2}\sum_a^l(z-k)\Delta+H_o\frac{\Sigma y\Delta}{\Sigma\Delta}-20\frac{dx}{2}V_o\text{ - - - - - - - - (47)}$$

$$V_x=\begin{cases}V_o,\text{ when }x<a\\ V_o-1,\text{for a single unit load when }x>a\end{cases}\text{ - - - (48)}$$

61819—27†——2

$$H_x = H_o$$

$$M_x = M_o + \left[V_o z - \overset{z>k}{(z-k)} \right] \frac{dx}{2} - H_o y \quad (49)$$

$$N_x = H \cos \phi + V_x \sin \phi \quad (50)$$

$$V_t = 0 \quad (51)$$

$$H_t = 20 \frac{etE}{C} \quad (52)$$

$$M_t = -H_t \left(y - \frac{\Sigma y \Delta}{\Sigma \Delta} \right) \quad (53)$$

$$f_c = \frac{N}{A} \pm M \frac{h}{2I} \quad (54)$$

The term $\frac{\cos \phi}{A}$ in some of the equations includes the effect of the so-called axial or rib shortening stress. This stress is sometimes considered separately from the other stresses and sometimes is neglected entirely. If the rise of the arch is relatively large, the axial stress is small and little accuracy is sacrificed by neglecting it, but if the arch is flat its effect is considerable and it should not be neglected.

Since it is easily taken care of on the forms, there appears to be no good reason for neglecting it or separating it from the rest of the stress, so it will be left in the formula and on the forms in its correct place, and the axial stress will be included in all cases.

CALCULATIONS FOR A SYMMETRICAL ARCH

The method of making the calculations for a symmetrical arch can best be explained by working out an actual example, and each step will be taken up in order and explained in detail. All calculations are entered in Tables 1 to 9, forms for which can be provided on five sheets of letter-size paper, and these forms are grouped in the text as used by the authors. Numerical values and plus and minus signs, which are the same for all arches, are shown in boldface type. In actual practice, blank forms made by the white-print process and showing column headings and other information common to all symmetrical arches are used. The formulas to be used are those developed for symmetrical arches. The formulas and forms for unsymmetrical arches could be used, but most arches are symmetrical, and the work is much reduced by using the simplified rather than the general formulas.

Assume that a 60-foot arch is to be designed for a uniform live load of 125 pounds per square foot (concentrated loads may be used without difficulty) and that the arch must withstand temperature stresses caused by a rise in temperature of 30° F. or a drop of 40° F. The tentative proportions of the arch and its reinforcement are determined and half of the arch ring plotted on detail paper on a scale of 1 inch to 2 feet. The rather large scale is used because the stresses to be determined will depend on scaled dimensions from this drawing. A similar drawing on a smaller scale is made on letter-size paper to accompany the tables of calculations as illustrated in Figure 5 (sheet 1 of forms).

After plotting one-half of the arch ring, draw the axis of the arch ring, which is a curve lying halfway between the intrados and extrados. Draw a vertical line through the springing line of the intrados until it intersects the arch axis as shown in Figure 5. This point is the origin of coordinates and is referred to as the origin, or point O. Through the origin draw a horizontal line, and on this line divide the half span into 10 equal parts. The length of each part will be equal to dx. On sheet 2 (Tables 1, 2, and 3) of the forms record in the places indicated the values of l, dx and the rise, which is the vertical distance from the arch axis at the crown to the horizontal line drawn through the origin. At the center of each dx erect a perpendicular to intersect with the arch axis, and mark these intersections 1, 2, 3, etc. In working out this example maximum stresses will be determined at points 0, 3, 8, and 10½ and the abutment pressure on the foundation determined.

The general method of procedure is to place a load of unity successively at each of the points on the arch ring and compute coefficients which can be applied to the actual dead and live loads for determining moments, shears, and thrusts.

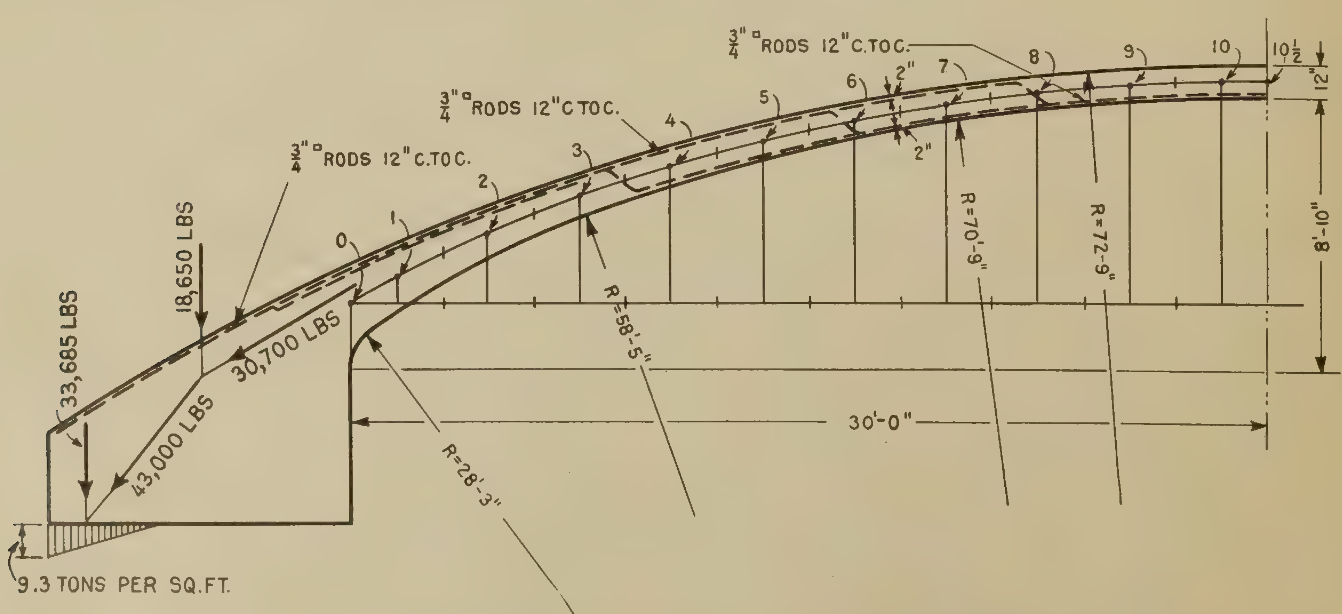


Fig. 5.—Sketch of Arch to Accompany Calculation Sheets

TABLE 1.—*Computations for* Δ

1	2	3	4	5	6	7	8	9	10	11
Points	h	h^3	$I_c=\frac{h^3}{12}$	$\frac{h}{2}$	$\frac{h}{2}-d'$	$\left(\frac{h}{2}-d'\right)^2$	$I_s=\left(\frac{h}{2}-d'\right)^2 nA_s$	$I=I_c+I_s$	ds	$\Delta=\frac{ds}{I}$
0	2.30	12.167	1.014	1.15	0.98	0.960	0.112	1.126	—	—
1	1.98	7.762	.647	.99	.82	.672	.079	.726	3.43	4.7
2	1.51	3.443	.287	.75	.58	.336	.039	.326	3.29	10.1
3	1.30	2.197	.183	.65	.48	.230	.013	.196	3.17	16.2
4	1.18	1.643	.137	.59	.42	.176	.010	.147	3.13	21.3
5	1.10	1.331	.111	.55	.38	.144	.008	.119	3.09	26.0
6	1.05	1.158	.096	.52	.35	.122	.007	.103	3.05	29.6
7	1.02	1.061	.088	.51	.34	.116	.007	.095	3.02	31.8
8	1.02	1.061	.088	.51	.34	.116	.007	.095	3.01	31.7
9	1.00	1.000	.083	.50	.33	.109	.006	.089	3.00	33.7
10	1.00	1.000	.083	.50	.33	.109	.006	.089	3.00	33.7
									$\frac{1}{2}\Sigma\Delta=$	238.8

TABLE 2.—*Computations for* V_o

Points	12	13	14	15	16	17	18
	$z-20$	$(z-20)\Delta$	$\Sigma Q=\sum_a^l (z-20)\Delta$	$\frac{1}{2}\sum_a^l(z-k)Q$	z^2	$z^2\Delta$	V_o
0	—	—	—	—	—	—	—
1	−19	89.3	+89.3	+20,091.6	1,522	7,153.4	1.00000
2	−17	−171.7	+261.0	+20,002.3	1,378	13,917.8	.99556
3	−15	−243.0	+504.0	+19,741.3	1,250	20,250.0	.98256
4	−13	−276.9	+780.9	+19,237.3	1,138	24,239.4	.95748
5	−11	−286.0	+1,066.9	+18,456.4	1,042	27,092.0	.91861
6	−9	−266.4	+1,333.3	+17,389.5	962	28,475.2	.86551
7	−7	−222.6	+1,555.9	+16,056.2	898	28,556.4	.79915
8	−5	−158.5	+1,714.4	+14,500.3	850	26,945.0	.72171
9	−3	−101.1	+1,815.5	+12,785.9	818	27,566.4	.63638
10	−1	−33.7	+1,849.2	+10,970.4	802	27,027.4	.54602
		−1,849.2	+10,970.4		$\Sigma z^2\Delta=$	231,223.2	

TABLE 3.—*Computations for* H_o

19	20	21	22	23	24	25	26	27	28	29	30
Points	y	Δ	$y\Delta$	$y-\frac{\Sigma y\Delta}{\Sigma\Delta}$	$\Delta\left(y-\frac{\Sigma y\Delta}{\Sigma\Delta}\right)$	$\Sigma Q=\sum_a^l\Delta\left(y-\frac{\Sigma y\Delta}{\Sigma\Delta}\right)$	$\frac{1}{2}\sum_a^l(z-k)Q$	$y\Delta\left(y-\frac{\Sigma y\Delta}{\Sigma\Delta}\right)$	$\cos\phi$	$\frac{\cos\phi}{A}$	H_o
0	0	—	—	—	—	—	—	—	0.838	—	—
1	0.88	4.7	4.136	−4.976	−23.387	+23.352	0.000	−20.581	.874	0.441	0.000
2	2.32	10.1	23.432	−3.536	−35.714	+59.066	−23.352	−82.856	.911	.603	+.062
3	3.51	16.2	56.862	−2.346	−38.005	+97.071	−82.418	−133.398	.946	.728	+.219
4	4.48	21.3	95.426	−1.376	−29.309	+126.380	−179.489	−131.304	.948	.804	+.478
5	5.29	26.0	137.540	−0.566	−14.716	+141.096	−305.869	−77.848	.971	.883	+.815
6	5.93	29.6	175.528	+0.074	+2.190	+138.906	−446.965	+12.987	.984	.938	+1.191
7	6.48	31.8	206.064	+0.624	+19.843	+119.063	−585.871	+128.583	.993	.974	+1.561
8	6.85	31.7	217.145	+0.994	+31.510	+87.553	−704.934	+215.844	.996	.976	+1.878
9	7.10	33.7	239.270	+1.244	+41.923	+45.630	−792.487	+297.653	1.000	1.000	+2.111
10	7.21	33.7	242.977	+1.354	+45.630	0.000	−838.117	+328.992	1.000	1.000	+2.232
		$\frac{1}{2}\Sigma y\Delta=$	1,398.378		0.000	838.117	$\frac{1}{2}\Sigma=$	+538.072	$\frac{1}{2}\Sigma=$	8.347	

Span $=l=60.00$ ft.

Rise $=7.22$ ft.

$$dx=\frac{l}{20}=3.00$$

$$\frac{1}{2}\,dx=1.50$$

$$\frac{\Sigma y\Delta}{\Sigma\Delta}=5.85585$$

$$F=\frac{1}{2}\Sigma z^2\Delta-200\,\Sigma\Delta=20{,}091.6$$

$$C=\frac{1}{dx}\sum_o^l y\Delta\left(y-\frac{\Sigma y\Delta}{\Sigma\Delta}\right)+\Sigma\frac{\cos.\ \phi}{A}=375.409$$

$$V_o=\frac{\frac{1}{2}\sum_a^l(z-k)(z-20)\Delta}{F}\qquad H_o=\frac{-\frac{1}{2}\sum_a^l(z-k)\Delta\left(y-\frac{\Sigma y\Delta}{\Sigma\Delta}\right)}{C}\qquad H_t=\frac{20\ etE}{C}=\frac{34560\,t}{C}=\frac{+2{,}762}{-3{,}682}\text{ for }t=\frac{+30}{-40}$$

$$\text{Col. }15=\frac{1}{2}\sum_a^l(z-k)\ (z-20)\Delta\qquad\text{Col. }26=\frac{1}{2}\sum_a^l(z-k)\ \Delta\left(y-\frac{\Sigma y\Delta}{\Sigma\Delta}\right)$$

Table 1.—Computations for Δ.—Scale accurately the thickness of the arch ring h at each point and set down the result in column 2. Columns 3 to 9, inclusive, are used in finding the moments of inertia of the sections of the arch ring at the various points. $I=I_c+I_s$, in which I is the total moment of inertia of the section about the axis of the arch, I_c the moment of inertia of the concrete, and I_s the moment of inertia of the steel. It is convenient to consider a strip of the arch ring 1 foot in width and scale all dimensions in feet, then $I_c=\frac{bh^3}{12}$, or, since $b=1$, $I_c=\frac{h^3}{12}$, which is tabulated in column 4.

$I_s=I_o+A_sj^2$ in which I_s is the moment of inertia of the steel about the axis of the arch, I_o is the moment of inertia of the steel about its own axis, A_s is the area of the steel, and j is the distance from the axis of the arch to the axis of the steel. Since the strength of steel is equivalent to n times that of the same area of concrete, and all of the dimensions are in feet, A_s in the above formula must be taken as equal to n times the total area of steel in square inches divided by 144.

$$n=\frac{E_s}{E_c}=15\qquad j=\frac{h}{2}-d'$$

in which d' is the distance from the surface of the concrete to the center of the steel. I_o is so small that it may be neglected.

Having computed I for each section and having the results tabulated in column 9, next scale ds for each section along the axis of the arch ring. The values of Δ are then computed. This should be carefully done because there is no check on the work and the accuracy of all of the rest of the columns will depend on the accuracy of column 11.

Special method of computation for terms involving $\frac{1}{2}\sum_a^l(z-k)$.—Column 12 (also 16 and 32) is the same for all arches and is already filled out on the printed form. In preparing the forms the values of z for the different points have been determined from the formula $x=z\,\frac{dx}{2}$. By inspection of Figure 1 it is evident that the value of z for point 1 is 1, for point 2 it is 3, for point 10 it is 19, etc.

The numerators of the formulas for V_o and H_o and one term of the formula for M_o contain a term which may be written in the form,

$$\frac{1}{2}\sum_a^l(z-k)\ Q$$

In each of these formulas Q has a different value, but the method of procedure in arriving at the value of the above expresssion is the same in each case and the term Q is used in order that a single explanation may serve for three cases.

In the formula for V_o, $Q=(z-20)\ \Delta$

In the formula for H_o, $Q=\Delta\ \left(y-\frac{\Sigma\ y\Delta}{\Sigma\Delta}\right)$

In the formula for M_o, $Q=\Delta$.

In each case Q depends solely on the properties of the arch and has the values Q_1, Q_2, etc., for the points along the arch ring.

We will consider first the formula for M_o because we have there the values of $Q=\Delta$ for the entire arch ring while in the other two cases we have the values of Q for only one-half of the arch ring. Column 35 of Table 4 is copied from column 11 of Table 1. Table 4, A, has been inserted to show the manner in which the values in columns 36 and 37 are secured. This table and Table 4, B, are not part of the forms and are inserted for purposes of explanation only. Column 36 of Table 4, A, shows that ΣQ for any point is the sum of values of Q for all points to the right of the point considered and is a matter of successive addition starting at the bottom of the column.

To understand the derivation of column 37 in Tables 4 and 4, A, it is necessary to study Table 4, B. In this table it is considered that a unit load is successively placed at each point and in each case the value of $\frac{1}{2}\overset{l}{\underset{a}{\Sigma}}(z-k)Q$ determined for each point to the right of the load. Values are not determined to the left of the load since the formula requires the values only where z is greater than k. At the point of load application z is equal to k and the expression becomes zero. The determinations of z and k for Table 4, B, are obvious from an examination of Figure 1. The value of $\frac{1}{2}\overset{l}{\underset{a}{\Sigma}}\ (z-k)\ Q$ for a unit load at any point is the sum of the column headed $\frac{1}{2}\ (z-k)\ Q$ and bearing the proper point designation. An inspection will show that the totals of these columns are the same as the quantities obtained in column 37 of Table 4, A, by the simple process of successive addition.

Table 2.—Computations for V_o.—We can now return to Table 2 of the forms and compute the values of V_o from the formula

$$V_o=\frac{\frac{1}{2}\overset{l}{\underset{a}{\Sigma}}(z-k)(z-20)\Delta}{\frac{1}{2}\Sigma z^2\Delta-200\Sigma\Delta}$$

In this formula Q, as used in the preceding explanation is equal to $(z-20)\Delta$. The values of $(z-20)$ are the same for all arches when the arch ring is divided into 20 parts so they are permanently printed in column 12 of the forms. The values of $(z-20)\Delta$ are computed by multiplying each term in column 11 by the corresponding term in column 12 and the results written in column 13. If Table 2 were made out for the entire arch ring the computation of columns 14 and 15 would be exactly as explained for columns 36 and 37 in Table 4, A, but due to the symmetry of the arch ring it is necessary to use only the points on one side of the crown.

The values of Δ are symmetrical. That is the value of Δ_{20} is the same as Δ_1, etc. The values of $(z-20)$ on the right-hand side of the arch are numerically the same as those on the left but with the opposite algebraic sign. Therefore the values of $Q=(z-20)\Delta$ on the right-hand half of the arch are the same numerically as those on the left but of opposite algebraic sign. Referring to Table 4, A, we see that the quantity in column 36 for point 10 is $(Q_{20}+Q_{19}\ \cdot\ \cdot\ \cdot\ Q_{11})$ which is equal to $-(Q_1+Q_2\ \cdot\ \cdot\ \cdot\ Q_{10})$. Therefore we find the sum of column 13 and write it opposite point 10 in column 14 but with opposite algebraic sign. We then continue the process of successive addition to the top of the column as previously explained.

A Filled-Spandrel Arch of the Type Illustrated by the Example

Referring to Table 4, A, we see that the quantity in column 37 opposite point 10 which corresponds to the bottom figure in column 15 of Table 2 is $(10Q_{20}+9Q_{19}\ \cdot\ \cdot\ \cdot\ Q_{11})$ and this is equal to the sum of column 14. Therefore we find the sum of column 14 and write it in the bottom space of column 15. Next we add the figure in column 14 opposite point 9 and set down the result in column 15 opposite point 9 and continue the process of successive addition to the top of the column. Examination of column 37 of Table 4, A, and column 14 of Table 2 will demonstrate the correctness of this procedure. As a partial check on the numerical work it should be noted that the top figure of column 14 is numerically the same as the top figure of column 13. Column 15 now contains the values of $\frac{1}{2}\Sigma(z-k)(z-20)\Delta$ for a unit load placed successively at each point on the left half of the arch ring.

The denominator of the equation for V_o; that is, $F=\frac{1}{2}\Sigma z^2\Delta-200\Sigma\Delta$, is independent of the loading and is easily computed. The values of z^2 are permanently printed in column 16 of the forms and for convenience the symmetrical values are combined; that is, for points 1 and 20, $z^2=(1^2+39^2)=1522$. For points 2 and 19 combined, $z^2=(3^2+37^2)=1378$, etc.

Column 17 is computed by multiplying each term in column 16 by the corresponding values of Δ in column 11. The sum of column 17 is equal to $\Sigma z^2\Delta$ and the sum of column 11 is equal to $\frac{1}{2}\Sigma\Delta$. The value of F is then computed and entered on the form. As a check on the numerical work it should be noted that the top figure of column 15 is equal to the value of F.

V_o can now be computed and tabulated in column 18 by dividing each term in column 15 by F or, since it is easier to multiply than to divide on most calculating machines, it will be quicker to find the reciprocal of F and multiply.

TABLE 4, A.—*Supplementary table to explain method of computing* $\frac{1}{2}\sum_a^l (z-k)\ Q$

Points	35 Q	36 $\sum_a^l Q$	37 $\frac{1}{2}\sum_a^l (z-k)\,Q$
1	Q_1	$Q_{20}+Q_{19} \ldots +Q_2$	$19\,Q_{20}+18\,Q_{19} \ldots +2\,Q_3+Q_2$
2	Q_2	$Q_{20}+Q_{19} \ldots +Q_3$	$18\,Q_{20}+17\,Q_{19} \ldots +2\,Q_4+Q_3$
3	Q_3	$Q_{20}+Q_{19} \ldots +Q_4$	$17\,Q_{20}+16\,Q_{19} \ldots +2\,Q_5+Q_4$
4	Q_4	$Q_{20}+Q_{19} \ldots +Q_5$	$16\,Q_{20}+15\,Q_{19} \ldots +2\,Q_6+Q_5$
5	Q_5	$Q_{20}+Q_{19} \ldots +Q_6$	$15\,Q_{20}+14\,Q_{19} \ldots +2\,Q_7+Q_6$
6	Q_6	$Q_{20}+Q_{19} \ldots +Q_7$	$14\,Q_{20}+13\,Q_{19} \ldots +2\,Q_8+Q_7$
7	Q_7	$Q_{20}+Q_{19} \ldots +Q_8$	$13\,Q_{20}+12\,Q_{19} \ldots +2\,Q_9+Q_8$
8	Q_8	$Q_{20}+Q_{19} \ldots +Q_9$	$12\,Q_{20}+11\,Q_{19} \ldots +2\,Q_{10}+Q_9$
9	Q_9	$Q_{20}+Q_{19} \ldots +Q_{10}$	$11\,Q_{20}+10\,Q_{19} \ldots +2\,Q_{11}+Q_{10}$
10	Q_{10}	$Q_{20}+Q_{19} \ldots +Q_{11}$	$10\,Q_{20}+9\,Q_{19} \ldots +2\,Q_{12}+Q_{11}$
11	Q_{11}	$Q_{20}+Q_{19} \ldots +Q_{12}$	$9\,Q_{20}+8\,Q_{19} \ldots +2\,Q_{13}+Q_{12}$
12	Q_{12}	$Q_{20}+Q_{19} \ldots +Q_{13}$	$8\,Q_{20}+7\,Q_{19} \ldots +2\,Q_{14}+Q_{13}$
13	Q_{13}	$Q_{20}+Q_{19} \ldots +Q_{14}$	$7\,Q_{20}+6\,Q_{19} \ldots +2\,Q_{15}+Q_{14}$
14	Q_{14}	$Q_{20}+Q_{19} \ldots +Q_{15}$	$6\,Q_{20}+5\,Q_{19} \ldots +2\,Q_{16}+Q_{15}$
15	Q_{15}	$Q_{20}+Q_{19} \ldots +Q_{16}$	$5\,Q_{20}+4\,Q_{19} \ldots +2\,Q_{17}+Q_{16}$
16	Q_{16}	$Q_{20}+Q_{19} \ldots +Q_{17}$	$4\,Q_{20}+3\,Q_{19} \ldots +2\,Q_{18}+Q_{17}$
17	Q_{17}	$Q_{20}+Q_{19}+Q_{18}$	$3\,Q_{20}+2\,Q_{19}+Q_{18}$
18	Q_{18}	$Q_{20}+Q_{19}$	$2\,Q_{20}+Q_{19}$
19	Q_{19}	Q_{20}	Q_{20}
20	Q_{20}	0	0

TABLE 4, B.—*Supplementary table to explain the method of computing* $\frac{1}{2}\sum_a^l (z-k)\,Q$

Points	z	Q	Load at point 20, $k=39$: $(z-k)$	$\frac{1}{2}(z-k)$	$\frac{1}{2}(z-k)Q$	Load at point 19, $k=37$: $(z-k)$	$\frac{1}{2}(z-k)$	$\frac{1}{2}(z-k)Q$	Load at point 18, $k=35$: $(z-k)$	$\frac{1}{2}(z-k)$	$\frac{1}{2}(z-k)Q$	Load at point 17, $k=33$: $(z-k)$	$\frac{1}{2}(z-k)$	$\frac{1}{2}(z-k)Q$	Load at point 16, $k=31$: $(z-k)$	$\frac{1}{2}(z-k)$	$\frac{1}{2}(z-k)Q$
1	1	Q_1															
2	3	Q_2															
3	5	Q_3															
4	7	Q_4															
5	9	Q_5															
6	11	Q_6															
7	13	Q_7															
8	15	Q_8															
9	17	Q_9															
10	19	Q_{10}															
11	21	Q_{11}															
12	23	Q_{12}															
13	25	Q_{13}															
14	27	Q_{14}															
15	29	Q_{15}															
16	31	Q_{16}												0	0	0
17	33	Q_{17}										0	0	0	2	1	Q_{17}
18	35	Q_{18}							0	0	0	2	1	Q_{18}	4	2	$2Q_{18}$
19	37	Q_{19}				0	0	0	2	1	Q_{19}	4	2	$2Q_{19}$	6	3	$3Q_{19}$
20	39	Q_{20}	0	0	0	2	1	Q_{20}	4	2	$2Q_{20}$	6	3	$3Q_{20}$	8	4	$4Q_{20}$

Points	z	Q	Load at point 15, $k=29$: $(z-k)$	$\frac{1}{2}(z-k)$	$\frac{1}{2}(z-k)Q$	Load at point 14, $k=27$: $(z-k)$	$\frac{1}{2}(z-k)$	$\frac{1}{2}(z-k)Q$	Load at point 13, $k=25$: $(z-k)$	$\frac{1}{2}(z-k)$	$\frac{1}{2}(z-k)Q$	Load at point 12, $k=23$: $(z-k)$	$\frac{1}{2}(z-k)$	$\frac{1}{2}(z-k)Q$	Load at point 11, $k=21$: $(z-k)$	$\frac{1}{2}(z-k)$	$\frac{1}{2}(z-k)Q$
1	1	Q_1															
2	3	Q_2															
3	5	Q_3															
4	7	Q_4															
5	9	Q_5															
6	11	Q_6															
7	13	Q_7															
8	15	Q_8															
9	17	Q_9															
10	19	Q_{10}															
11	21	Q_{11}													0	0	0
12	23	Q_{12}										0	0	0	2	1	$1Q_{12}$
13	25	Q_{13}							0	0	0	2	1	Q_{13}	4	2	$2Q_{13}$
14	27	Q_{14}				0	0	0	2	1	Q_{14}	4	2	$2Q_{14}$	6	3	$3Q_{14}$
15	29	Q_{15}	0	0	0	2	1	Q_{15}	4	2	$2Q_{15}$	6	3	$3Q_{15}$	8	4	$4Q_{15}$
16	31	Q_{16}	2	1	Q_{16}	4	2	$2Q_{16}$	6	3	$3Q_{16}$	8	4	$4Q_{16}$	10	5	$5Q_{16}$
17	33	Q_{17}	4	2	$2Q_{17}$	6	3	$3Q_{17}$	8	4	$4Q_{17}$	10	5	$5Q_{17}$	12	6	$6Q_{17}$
18	35	Q_{18}	6	3	$3Q_{18}$	8	4	$4Q_{18}$	10	5	$5Q_{18}$	12	6	$6Q_{18}$	14	7	$7Q_{18}$
19	37	Q_{19}	8	4	$4Q_{19}$	10	5	$5Q_{19}$	12	6	$6Q_{19}$	14	7	$7Q_{19}$	16	8	$8Q_{19}$
20	39	Q_{20}	10	5	$5Q_{20}$	12	6	$6Q_{20}$	14	7	$7Q_{20}$	16	8	$8Q_{20}$	18	9	$9Q_{20}$

Points	z	Q	Load at point 10, $k=19$: $(z-k)$	$\frac{1}{2}(z-k)$	$\frac{1}{2}(z-k)Q$	Load at point 9, $k=17$: $(z-k)$	$\frac{1}{2}(z-k)$	$\frac{1}{2}(z-k)Q$	Load at point 8, $k=15$: $(z-k)$	$\frac{1}{2}(z-k)$	$\frac{1}{2}(z-k)Q$	Load at point 7, $k=13$: $(z-k)$	$\frac{1}{2}(z-k)$	$\frac{1}{2}(z-k)Q$	Load at point 6, $k=11$: $(z-k)$	$\frac{1}{2}(z-k)$	$\frac{1}{2}(z-k)Q$
1	1	Q_1															
2	3	Q_2															
3	5	Q_3															
4	7	Q_4															
5	9	Q_5															
6	11	Q_6													0	0	0
7	13	Q_7										0	0	0	2	1	Q_7
8	15	Q_8							0	0	0	2	1	Q_8	4	2	$2\ Q_8$
9	17	Q_9				0	0	0	2	1	Q_9	4	2	$2\ Q_9$	6	3	$3\ Q_9$
10	19	Q_{10}	0	0	0	2	1	Q_{10}	4	2	$2Q_{10}$	6	3	$3Q_{10}$	8	4	$4Q_{10}$
11	21	Q_{11}	2	1	Q_{11}	4	2	$2Q_{11}$	6	3	$3Q_{11}$	8	4	$4Q_{11}$	10	5	$5Q_{11}$
12	23	Q_{12}	4	2	$2Q_{12}$	6	3	$3Q_{12}$	8	4	$4Q_{12}$	10	5	$5Q_{12}$	12	6	$6Q_{12}$
13	25	Q_{13}	6	3	$3Q_{13}$	8	4	$4Q_{13}$	10	5	$5Q_{13}$	12	6	$6Q_{13}$	14	7	$7Q_{13}$
14	27	Q_{14}	8	4	$4Q_{14}$	10	5	$5Q_{14}$	12	6	$6Q_{14}$	14	7	$7Q_{14}$	16	8	$8Q_{14}$
15	29	Q_{15}	10	5	$5Q_{15}$	12	6	$6Q_{15}$	14	7	$7Q_{15}$	16	8	$8Q_{15}$	18	9	$9Q_{15}$
16	31	Q_{16}	12	6	$6Q_{16}$	14	7	$7Q_{16}$	16	8	$8Q_{16}$	18	9	$9Q_{16}$	20	10	$10Q_{16}$
17	33	Q_{17}	14	7	$7Q_{17}$	16	8	$8Q_{17}$	18	9	$9Q_{17}$	20	10	$10Q_{17}$	22	11	$11Q_{17}$
18	35	Q_{18}	16	8	$8Q_{18}$	18	9	$9Q_{18}$	20	10	$10Q_{18}$	22	11	$11Q_{18}$	24	12	$12Q_{18}$
19	37	Q_{19}	18	9	$9Q_{19}$	20	10	$10Q_{19}$	22	11	$11Q_{19}$	24	12	$12Q_{19}$	26	13	$13Q_{19}$
20	39	Q_{20}	20	10	$10Q_{20}$	22	11	$11Q_{20}$	24	12	$12Q_{20}$	26	13	$13Q_{20}$	28	14	$14Q_{20}$

Points	z	Q	Load at point 5, $k=9$: $(z-k)$	$\frac{1}{2}(z-k)$	$\frac{1}{2}(z-k)Q$	Load at point 4, $k=7$: $(z-k)$	$\frac{1}{2}(z-k)$	$\frac{1}{2}(z-k)Q$	Load at point 3, $k=5$: $(z-k)$	$\frac{1}{2}(z-k)$	$\frac{1}{2}(z-k)Q$	Load at point 2, $k=3$: $(z-k)$	$\frac{1}{2}(z-k)$	$\frac{1}{2}(z-k)Q$	Load at point 1, $k=1$: $(z-k)$	$\frac{1}{2}(z-k)$	$\frac{1}{2}(z-k)Q$
1	1	Q_1													0	0	0
2	3	Q_2										0	0	0	2	1	Q_2
3	5	Q_3							0	0	0	2	1	Q_3	4	2	$2\ Q_3$
4	7	Q_4				0	0	0	2	1	Q_4	4	2	$2\ Q_4$	6	3	$3\ Q$
5	9	Q_5	0	0	0	2	1	Q_5	4	2	$2\ Q_5$	6	3	$3\ Q_5$	8	4	$4\ Q_5$
6	11	Q_6	2	1	Q_6	4	2	$2\ Q_6$	6	3	$3\ Q_6$	8	4	$4\ Q_6$	10	5	$5\ Q_6$
7	13	Q_7	4	2	$2\ Q_7$	6	3	$3\ Q_7$	8	4	$4\ Q_7$	10	5	$5\ Q_7$	12	6	$6\ Q_7$
8	15	Q_8	6	3	$3\ Q_8$	8	4	$4\ Q_8$	10	5	$5\ Q_8$	12	6	$6\ Q_8$	14	7	$7\ Q_8$
9	17	Q_9	8	4	$4\ Q_9$	10	5	$5\ Q_9$	12	6	$6\ Q_9$	14	7	$7\ Q_9$	16	8	$8\ Q_9$
10	19	Q_{10}	10	5	$5Q_{10}$	12	6	$6Q_{10}$	14	7	$7Q_{10}$	16	8	$8Q_{10}$	18	9	$9Q_{10}$
11	21	Q_{11}	12	6	$6Q_{11}$	14	7	$7Q_{11}$	16	8	$8Q_{11}$	18	9	$9Q_{11}$	20	10	$10Q_{11}$
12	23	Q_{12}	14	7	$7Q_{12}$	16	8	$8Q_{12}$	18	9	$9Q_{12}$	20	10	$10Q_{12}$	22	11	$11Q_{12}$
13	25	Q_{13}	16	8	$8Q_{13}$	18	9	$9Q_{13}$	20	10	$10Q_{13}$	22	11	$11Q_{13}$	24	12	$12Q_{13}$
14	27	Q_{14}	18	9	$9Q_{14}$	20	10	$10Q_{14}$	22	11	$11Q_{14}$	24	12	$12Q_{14}$	26	13	$13Q_{14}$
15	29	Q_{15}	20	10	$10Q_{15}$	22	11	$11Q_{15}$	24	12	$12Q_{15}$	26	13	$13Q_{15}$	28	14	$14Q_{15}$
16	31	Q_{16}	22	11	$11Q_{16}$	24	12	$12Q_{16}$	26	13	$13Q_{16}$	28	14	$14Q_{16}$	30	15	$15Q_{16}$
17	33	Q_{17}	24	12	$12Q_{17}$	26	13	$13Q_{17}$	28	14	$14Q_{17}$	30	15	$15Q_{17}$	32	16	$16Q_{17}$
18	35	Q_{18}	26	13	$13Q_{18}$	28	14	$14Q_{18}$	30	15	$15Q_{18}$	32	16	$16Q_{18}$	34	17	$17Q_{18}$
19	37	Q_{19}	28	14	$14Q_{19}$	30	15	$15Q_{19}$	32	16	$16Q_{19}$	34	17	$17Q_{19}$	36	18	$18Q_{19}$
20	39	Q_{20}	30	15	$15Q_{20}$	32	16	$16Q_{20}$	34	17	$17Q_{20}$	36	18	$18Q_{20}$	38	19	$19Q_{20}$

Table 3.—Computations for H_o.—Table 3 is used in computing H_o from the formula

$$H_o=\frac{-\frac{1}{2}\sum_{a}^{l}\left(z-k\right)\Delta\left(y-\frac{\Sigma y\Delta}{\Sigma\Delta}\right)}{C}$$

In this formula the denominator C is equal to

$$\frac{1}{dx}\sum_{0}^{l}y\Delta\left(y-\frac{\Sigma y\Delta}{\Sigma\Delta}\right)+\sum_{0}^{l}\frac{\cos\phi}{A}$$

and is independent of the loading. The numerator depends on the position of the unit load and is computed in a way similar to that used for the formula for V_o. Values of y are scaled from the drawing and tabulated in column 20. For convenience the values of Δ are copied in column 21. The values of $y\Delta$ are found by multiplying, and then the value of $\frac{\Sigma y\Delta}{\Sigma\Delta}$ may be found by dividing the sum of column 22 by the sum of column 11. Column 23 is computed by subtracting the constant value of $\frac{\Sigma y\Delta}{\Sigma\Delta}$ from each value of y. Then column 24 may be computed by multiplying each term in column 23 by the corresponding value of Δ. The sum of column 24 should be zero. If it is not it is due to some inaccuracy which should be found before going further. The error will be in column 21, 22, 23, or 24. Of course allowance should be made for the fact that decimal places were dropped in the value of $\frac{\Sigma y\Delta}{\Sigma\Delta}$ and that inaccuracy is carried into columns 24 and 25. For that reason the sum of column 24 will usually not check exactly, but it is not difficult to determine if the inaccuracy is from that cause or a mistake.

Columns 25 and 26 are computed from column 24 in exactly the same manner as columns 14 and 15 were computed from column 13.

The bottom figure in column 25 is equal to the sum of the figures of column 24 for the right-hand half of the arch, which are not shown, but they are the same as the figures which are shown and their sum is also zero. To this zero we add the last figure shown in column 24 and write the result in the next space above in column 25 and continue the process of successive addition to the top of the column. The top figure of column 25 should be approximately equal numerically to the top figure of column 24.

The bottom figure in column 26 is the sum of the figures in the half of column 25 which is not shown plus the figure opposite point 10, but since the latter figure is always zero it makes no difference in the sum. This sum is equal to minus the sum of the figures shown. This may be understood from consideration of Table 4, A. Thus we find the sum of the figures shown in column 25 and write it in the bottom space of column 26. Then we add to it the figure opposite point 9 in column 25 and continue the process of successive addition to the top of the column, observing the correct algebraic sign. Then all of the figures in column 26 will be negative and the top figure will be zero.

The denominator of the equation for H_o that is

$$C=\frac{1}{dx}\,\Sigma y\Delta\left(y-\frac{\Sigma y\Delta}{\Sigma\Delta}\right)+\Sigma\,\frac{\cos\phi}{A}$$

is independent of the loading and is computed in columns 27, 28, and 29 as indicated by the headings of the columns. Cos ϕ is tabulated in column 28. It might be computed from the formula $\cos\phi=\frac{dx}{ds}$, but since $\cos\phi$ varies so little for small angles it will be found more accurate to compute $\sin\phi$ from dimensions scaled on the drawing and then find the corresponding values of $\cos\phi$ in a table of trigonometric functions.

Column 29 is computed by dividing each term in column 28 by the area A. Since a strip of the arch ring 1 foot wide is being considered, A is equal to h plus n times the area of steel in square feet. Since the area of steel in square feet is small and the effect of $\cos\phi$ is relatively small, it is sufficiently accurate to assume, for this purpose, that A is equal to h. C, the denominator of the equation for H_o, is found by dividing the sum of column 27 by ½ dx and adding twice the sum of column 29. The value of C should be set down in the space provided on the same sheet with Table 3. H_o may now be computed for a load of unity at each point by dividing each term of column 26 by C and the results are tabulated in column 30. Note that the algebraic sign is changed because the formula for H_o is preceded by a minus sign.

Table 4.—Computation for M_o.—M_o is computed in Table 4 from the formula

$$M_o=H_o.\frac{\Sigma y\Delta}{\Sigma\Delta}-20\,\frac{dx}{2}\,V_o+\frac{dx}{\Sigma\Delta}\times\frac{1}{2}\sum_{a}^{l}\left(z-k\right)\Delta$$

The values of z are permanently printed in column 32. The values of H_o, V_o and Δ have been previously found and for convenience are copied in columns 33, 34, and 35. The values of H_o and Δ are symmetrical so the values found for the left-hand half of the arch ring may be copied for the right-hand half. The values of V_o for the right half of the arch may be found by subtracting each value for the left half from unity.

Column 36 is computed by summing up column 35, starting with zero in the bottom space for point 1′. Column 37 is computed by summing up column 36 in the same way starting with zero at the bottom opposite point 1′ as previously explained. Point 1′ for symmetrical arches is the same as point 20 used in the explanation. Column 38 is computed by multiplying each term in column 37 by $\frac{dx}{\Sigma\Delta}$. Column 39 is computed by multiplying each value of H_o by $\frac{\Sigma y\Delta}{\Sigma\Delta}$. Column 40 is computed by multiplying each value of V_o by $-20\,\frac{dx}{2}$. The sum of column 39 should equal the sum of column 33 multiplied by $\frac{\Sigma y\Delta}{\Sigma\Delta}$ and the sum of column 40 should equal $-100\,dx$. M_o in column 41 is computed by taking the algebraic sum of the corresponding terms in columns 38, 39, and 40. The algebraic sum of column 41 should equal the algebraic sum of columns 38, 39, and 40.

TABLE 4.—*Computations for M_o*

31	32	33	34	35	36	37	38	39	40	41
Points	z	H_o	V_o	Δ	$\sum_a^l \Delta$	$\frac{1}{2}\sum_a^l (z-k)\Delta$	$\frac{dx}{\Sigma\Delta}$Col. 37	$\frac{\Sigma y\Delta}{\Sigma\Delta}$	$-20\frac{dx}{2}V_o$	M_o
0	0									
1	1	0.000	1.0000	4.7	472.9	4,537.2	28.500	0.000	− 30.000	− 1.500
2	3	.062	.99556	10.1	462.8	4,064.3	25.529	.363	− 29.867	− 3.975
3	5	.219	.98256	16.2	446.6	3,601.5	22.622	1.282	− 29.477	− 5.573
4	7	.478	.95748	21.3	425.3	3,154.9	19.817	2.799	− 28.724	− 6.108
5	9	.815	.91861	26.0	399.3	2,729.6	17.146	4.772	− 27.558	− 5.640
6	11	1.191	.86551	29.6	369.7	2,330.3	14.637	6.974	− 25.965	− 4.354
7	13	1.561	.79915	31.8	337.9	1,960.6	12.315	9.141	− 23.974	− 2.518
8	15	1.878	.72171	31.7	306.2	1,622.7	10.193	10.997	− 21.651	− 0.461
9	17	2.111	.63638	33.7	272.5	1,316.5	8.269	12.362	− 19.091	+ 1.540
10	19	2.232	.54602	33.7	238.8	1,044.0	6.558	13.070	− 16.381	+ 3.247
10′	21	2.232	.45398	33.7	205.1	805.2	5.058	13.070	− 13.619	+ 4.509
9′	23	2.111	.36362	33.7	171.4	600.1	3.769	12.362	− 10.909	+ 5.222
8′	25	1.878	.27829	31.7	139.7	428.7	2.693	10.997	− 8.349	+ 5.341
7′	27	1.561	.20085	31.8	107.9	289.0	1.815	9.141	− 6.026	+ 4.930
6′	29	1.191	.13449	29.6	78.3	181.1	1.137	6.974	− 4.035	+ 4.076
5′	31	.815	.08139	26.0	52.3	102.8	.646	4.772	− 2.442	+ 2.976
4′	33	.478	.04252	21.3	31.0	50.5	.317	2.799	− 1.276	+ 1.840
3′	35	.219	.01744	16.2	14.8	19.5	.122	1.282	− .523	+ .881
2′	37	.062	.00444	10.1	4.7	4.7	.030	.363	− .133	+ .260
1′	39	0.000	0.0000	4.7	0.0	0.0	0.000	0.000	0.000	0.000
0′	40									
		21.094	10.0000				181.173	123.520	− 300.000	+ 4.693

$$M_o=\frac{dx}{\Sigma\Delta}\frac{1}{2}\sum_a^l(z-k)\Delta+H_o\frac{\Sigma y\Delta}{\Sigma\Delta}-20\frac{dx}{2}V_o$$

$$\frac{dx}{\Sigma\Delta}=.0062814 \qquad -20\frac{dx}{2}=-30.0$$

For check:

$$\frac{\Sigma y\Delta}{\Sigma\Delta}\Sigma\text{Col. }33=123.523=\Sigma\text{Col. }39.$$

$$-20\frac{dx}{2}\Sigma\text{Col. }34=-300.0=\Sigma\text{Col. }40.$$

$$\Sigma\text{Col. }38+\Sigma\text{Col. }39+\Sigma\text{Col. }40=+4.693=\Sigma\text{Col. }41.$$

Tables 5, 6, and 7.—Computation of bending moments at points 3, 8, and 10½.—We now have the values of H_o, V_o, and M_o for a load of unity placed at any point on the arch. It must now be decided at which other points the stresses are desired. Usually three other points are sufficient, and they should be selected with care in order to get the critical points. One should be at or near the crown, one near the haunch, and probably the best place for the third is between the haunch and the left abutment. Points 3 and 8 are chosen because they are near changes in the reinforcement; and point 10½, being the crown, is also chosen.

Columns 46 to 50, inclusive, are used in finding the moment at point 3, from the formula $M_3=M_o-H_oy+m_3$. In computing the moment at any particular point, z and y are constant, while k varies with the position of the unit load. We now desire the moment at a particular point caused by a load at each point. For point 3 the value of z is 5 and the value of y can be found in column 20. These figures should be set down in the spaces indicated in the heading for columns 46 to 50. For convenience H_o, V_o, and M_o are copied in columns 43, 44, and 45. The values of m_x are computed from the formula $m_x=\left[V_oz-(z-k)\right]\frac{dx}{2}$ where k is less than z, and from the formula $m_x=V_oz\frac{dx}{2}$ where k is greater than z. Column 46 is computed by multiplying the values of V_o by z ($z=5$ for point 3). It is only necessary to fill out column 46 for a load at those points where

TABLE 5.—*Computations for M_3*

42	43	44	45	46	47	48	49	50
				Point 3, $z_3=5$; $y_3=3.51$				
Points	H_o	V_o	M_o	V_oz_3	$V_oz_3-(z_3-k)$	m_3	$-H_oy_3$	M_3
0								
1	0.000	1.00000	−1.500	5.000	1.0000	1.500	0.000	0.000
2	.062	.99556	−3.975	4.978	2.978	4.467	− .218	+ .274
3	.219	.98256	−5.573			7.369	− .769	+1.027
4	.478	.95748	−6.108			7.181	− 1.678	− .605
5	.815	.91861	−5.640			6.890	− 2.861	−1.611
6	1.191	.86551	−4.354			6.491	− 4.180	−2.043
7	1.561	.79915	−2.518			5.994	− 5.479	−2.003
8	1.878	.72171	−0.461			5.413	− 6.592	−1.640
9	2.111	.63638	+1.540			4.773	− 7.409	−1.096
10	2.232	.54602	+3.247			4.095	− 7.834	− .492
10′	2.232	.45398	+4.509			3.405	− 7.834	+ .080
9′	2.111	.36362	+5.222			2.727	− 7.409	+ .540
8′	1.878	.27829	+5.341			2.087	− 6.592	+ .836
7′	1.561	.20085	+4.930			1.506	− 5.479	+ .957
6′	1.191	.13449	+4.076			1.009	− 4.180	+ .905
5′	.815	.08139	+2.976			.610	− 2.861	+ .725
4′	.478	.04252	+1.840			.319	− 1.678	+ .481
3′	.219	.01744	+ .881			.131	− .769	+ .243
2′	.062	.00444	+ .260			.033	− .218	+ .075
1′	0.000	0.00000	0.000	0.000	0.000	0.000	0.000	0.000
0′								
	21.094	10.00000	+4.693			66,000	−74.040	−3.347

TABLE 6.—*Computations for M_8*

Points	51	52	53	54	55
	Point 8, $z_8=15$; $y_8=6.85$				
	V_oz_8	$V_oz_8-(z_8-k)$	m_8	$-H_oy_8$	M_8
0					
1	15.000	1.0000	1.500	0.000	0.000
2	14.933	2.933	4.400	− .425	.000
3	14.738	4.738	7.107	− 1.500	+ .034
4	14.362	6.362	9.543	− 3.274	+ .161
5	13.779	7.779	11.669	− 5.583	+ .446
6	12.983	8.983	13.475	− 8.158	+ .963
7	11.987	9.987	14.981	−10.693	+1.770
8			16.238	−12.864	+2.913
9			14.319	−14.460	+1.399
10			12.285	−15.289	+ .243
10′			10.215	−15.289	− .565
9′			8.181	−14.460	−1.057
8′			6.262	−12.864	−1.261
7′			4.519	−10.693	−1.244
6′			3.026	− 8.158	−1.056
5′			1.831	− 5.583	− .776
4′			.957	− 3.274	− .477
3′			.392	− 1.500	− .227
2′			.100	− .425	− .065
1′	0.000	0.000	0.000	0.000	0.000
0′					
			141.000	−144.492	+1.201

TABLE 7.—*Computations for $M_{10\frac{1}{2}}$*

Points	56	57	58	59	60
	Point 10½, $z_{10\,1/2}=20$; $y_{10\,1/2}=7.21$				
	$V_oz_{10\,1/2}$	$V_oz_{10\,1/2}-(z_{10\,1/2}-k)$	$m_{10\,1/2}$	$-H_oy_{10\,1/2}$	$M_{10\,1/2}$
0					
1	20.000	1.000	1.500	0.000	0.000
2	19.911	2.911	4.367	− .447	− .055
3	19.651	4.651	6.976	− 1.579	− .176
4	19.150	6.150	9.225	− 3.446	− .329
5	18.372	7.372	11.058	− 5.876	− .458
6	17.310	8.310	12.465	− 8.587	− .476
7	15.983	8.983	13.475	− 11.255	− .298
8	14.434	9.434	14.151	− 13.540	+ .150
9	12.728	9.728	14.592	− 15.220	+ .912
10	10.920	9.920	14.880	− 16.093	+ 2.034
10′			13.619	− 16.093	+ 2.035
9′			10.909	− 15.220	+ .911
8′			8.349	− 13.540	+ .150
7′			6.025	− 11.255	− .300
6′			4.035	− 8.587	− .476
5′			2.442	− 5.876	− .458
4′			1.276	− 3.446	− .330
3′			.523	− 1.579	− .175
2′			.133	− .447	− .054
1′	0.000	0.000	0.000	0.000	0.000
0′					
			150.000	−152.086	+ 2.607

$$M_x=M_o+m_x-H_oy$$

$$m_x=\left[V_oz_x-(z_x-k)\right]^{z>k}\frac{dx}{2}$$

For check:

Σ Col. 45+Σ Col. 48+Σ Col .49=Σ Col. 50.
Σ Col. 45+Σ Col. 53+Σ Col. 54=Σ Col. 55.
Σ Col. 45+Σ Col. 58+Σ Col. 59=Σ Col. 60.

k is less than z. Column 47 is computed by subtracting $(z-k)$ from the figures in column 46. The values of m_x may now be computed for a load at those points where k is less than z by multiplying the figures in column 47 by $\frac{dx}{2}$ and for the remaining points by multiplying the values of V_o by $z\frac{dx}{2}$.

The value of $-H_oy$ in column 49 is computed by multiplying each value of H_o by $-y$ as given in the heading. The bending moment $M_3 = M_o + m_3 - H_oy$ is now computed by adding algebraically the corresponding values of columns 45, 48, and 49 and the results tabulated in column 50. The bending moments at the two other points are computed in the same way, using columns 51 to 60, inclusive. We now have in the tabulation which includes Tables 5, 6, and 7 (sheet 4 of assembly of data) the vertical shear, the horizontal thrust, and the bending moment at point 0 and the bending moment at three other points caused by a load of unity at each of the points of the arch ring.

Table 8.—Computations for the dead-load moments, thrust, and shears.—The values of H_o, V_o, M_o and the unit-load bending moments at the points 3, 8, and 10½ are copied in columns 62 to 67 of Table 8. The dead load which is applied at each point is then computed and tabulated in column 68. This may be conveniently tabulated in a supplementary table as follows: The dead load at any point is equal to $150hds + 110dx\ h_f$, in which the weight of the concrete is assumed to be 150 pounds per cubic foot and the weight of the earth fill 110 pounds per cubic foot. Values of h and ds are found in columns 2 and 10, respectively, of Table 1, and h_f is the depth of the fill above the arch ring and dx for this arch is 3 feet.

Computation of dead load

Point	$h\,ds$	$150\,h\,ds$	h_f	$330\,h_f$	Dead load
					Pounds
1	6.79	1,020	6.00	1,980	3,000
2	4.96	744	4.80	1,585	2,329
3	4.11	616	3.82	1,260	1,876
4	3.69	553	3.00	990	1,543
5	3.40	510	2.25	742	1,252
6	3.20	480	1.70	560	1,040
7	3.08	462	1.20	396	858
8	3.04	456	.90	297	753
9	3.00	450	.70	232	682
10	3.00	450	.67	220	670

Column 69 to 74, inclusive, are computed by multiplying the dead load in column 68 by the proper figure from columns 62 to 67, inclusive. The algebraic sums of columns 69, 70, and 71 are the horizontal thrust, vertical shear, and bending moment, respectively, at point 0 resulting from the dead load. The algebraic sums of columns 72, 73, and 74 are the dead-load bending moments at points 3, 8, and 10½.

Table 9—Computation of stresses.—Column 75 in Table 9 is now to be filled out. The values of cos ϕ for each point desired are taken from column 28. The values of sin ϕ may be found by scaling the drawing. Since the stresses are desired in pounds per square inch, and all previous dimensions have been in feet, the area in column 75 should be 144 times h (taken from column 2) plus n times the area of steel in square inches. The values of $\frac{h}{2I}$ are computed from columns 2 and 9 and this result should be divided by 144 to get results in pounds per square inch. The values of M, H, and V for the dead load are taken from the sums of columns 69 to 74, inclusive, and set down in the proper places in

TABLE 8.—*Computations of H, V, and M for dead load*

61	62	63	64	65	66	67	68	69	70	71	72	73	74
Point	Unit loads						D. L.	Dead load					
	H_o	V_o	M_o	M_3	M_8	$M_{10\frac{1}{2}}$		H_o	V_o	M_o	M_3	M_8	$M_{10\frac{1}{2}}$
1	0	1.00000	− 1.500	0	0	0	3,000	0	3,000	− 4,500	0	0	0
2	.062	.99556	− 3.975	+ .274	+ .000	− .055	2,330	144	2,320	− 9,262	+ 638	0	− 128
3	.219	.98256	− 5.573	+ 1.027	+ .034	− .176	1,880	412	1,847	− 10,477	+ 1,931	+ 64	− 331
4	.478	.95748	− 6.108	− .605	+ .161	− .329	1,540	736	1,475	− 9,406	− 932	+ 248	− 507
5	.815	.91861	− 5.640	− 1.611	+ .446	− .458	1,250	1,019	1,148	− 7,050	− 2,014	+ 558	− 573
6	1.191	.86551	− 4.354	− 2.043	+ .963	− .476	1,040	1,239	900	− 4,528	− 2,125	+ 1,002	− 495
7	1.561	.79915	− 2.518	− 2.003	+ 1.770	− .298	860	1,342	687	− 2,165	− 1,723	+ 1,522	− 256
8	1.878	.72171	− 0.461	− 1.640	+ 2.913	+ .150	750	1,409	541	− 346	− 1,230	+ 2,185	+ 113
9	2.111	.63638	+ 1.540	− 1.096	+ 1.399	+ .912	680	1,435	433	+ 1,047	− 745	+ 951	+ 620
10	2.232	.54602	+ 3.247	− .492	+ .243	+ 2.034	670	1,495	366	+ 2,175	− 330	+ 163	+ 1,363
10′	2.232	.45398	+ 4.509	+ .080	− .565	+ 2.035	670	1,495	304	+ 3,021	+ 54	− 379	+ 1,363
9′	2.111	.36362	+ 5.222	+ .540	− 1.057	+ .911	680	1,435	247	+ 3,551	+ 367	− 719	+ 619
8′	1.878	.27829	+ 5.341	+ .836	− 1.261	+ .150	750	1,409	209	+ 4,006	+ 627	− 946	+ 113
7′	1.561	.20085	+ 4.930	+ .957	− 1.244	− .300	860	1,342	173	+ 4,240	+ 823	− 1,070	− 258
6′	1.191	.13449	+ 4.076	+ .905	− 1.056	− .476	1,040	1,239	140	+ 4,239	+ 941	− 1,098	− 495
5′	.815	.08139	+ 2.976	+ .725	− .776	− .458	1,250	1,019	102	+ 3,720	+ 906	− 970	− 573
4′	.478	.04252	+ 1.840	+ .481	− .477	− .330	1,540	736	65	+ 2,834	+ 741	− 735	− 508
3′	.219	.01744	+ .881	+ .243	− .227	− .175	1,880	412	33	+ 1,656	+ 457	− 427	− 329
2′	.062	.00444	+ .260	+ .075	− .065	− .054	2,330	144	10	+ 606	+ 175	− 151	− 126
1′	0	0	0	0	0	0	3,000	0	0	0	0	0	0
	21.094	10.0000	+ 34.822	+ 6.143	+ 7.929	+ 6.192	28,000	18,462	14,000	+ 31,095	+ 7,660	+ 6,693	+ 4,191
			− 30.129	− 9.490	− 6.728	− 3.585				− 47,734	− 9,099	− 6,495	− 4,579
			+ 4.693	− 3.347	+ 1.201	+ 2.607				− 16,639	− 1,439	+ 198	− 388

$$M_t = -H_t\left(y - \frac{\Sigma y\Delta}{\Sigma\Delta}\right)$$

At point 0, $M_t = -\begin{Bmatrix}+2{,}762\\-3{,}682\end{Bmatrix}\times(-5.856) = \begin{Bmatrix}+16{,}174\\-21{,}562\end{Bmatrix}$

At point 3, $M_t = -\begin{Bmatrix}+2{,}762\\-3{,}682\end{Bmatrix}\times(-2.346) = \begin{Bmatrix}+\ 6{,}480\\-\ 8{,}638\end{Bmatrix}$

At point 8, $M_t = -\begin{Bmatrix}+2{,}762\\-3{,}682\end{Bmatrix}\times(+0.994) = \begin{Bmatrix}-\ 2{,}745\\+\ 3{,}660\end{Bmatrix}$

At point 10½, $M_t = -\begin{Bmatrix}+2{,}762\\-3{,}682\end{Bmatrix}\times(+1.364) = \begin{Bmatrix}-\ 3{,}470\\+\ 4{,}985\end{Bmatrix}$

TABLE 9.—*Computation of maximum stresses*

75	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90
										Extrados	Intrados	Maximum stresses			
		H	M	V	$H \cos \phi$	$V \sin \phi$	N	$\frac{N}{A}$	$M\frac{h}{2I}$	$\frac{N}{A}+M\frac{h}{2I}$	$\frac{N}{A}-M\frac{h}{2I}$	Extrados		Intrados	
												+	−	+	−
Point 0	D. L.	+ 18,462	− 16,639	14,000	+ 15,471	7,672	+ 23,142	+ 66	− 118	− 52	+ 184	− 52	− 52	+ 184	+ 184
sin ϕ=.548.	+ C. L. L.	+	+		+		+	+	+						
cos ϕ=.838.	+ U. L. L.	+ 5,584	+ 13,058	1,035	+ 4,679	567	+ 5,246	+ 15	+ 93	+ 108	− 78	+ 108			− 78
Area=348.	− C. L. L.	+	−		+		+	+	−						
	− U L. L.	+ 2,326	− 11,298	2,715	+ 1,949	1,488	+ 3,437	+ 10	− 80	− 70	+ 90		− 70	+ 90	
$\frac{h}{2I}$=.00709.	+ T.	+ 2,762	+ 16,174	0	+ 2,315	0	+ 2,315	+ 7	+ 115	+ 122	− 108	+ 122			− 108
	− T.	− 3,682	− 21,562	0	− 3,086	0	− 3,086	− 9	− 153	− 162	+ 144		− 162	+ 144	
									Total excluding temperature				− 122	+ 274	
											Total	+ 178	− 284	+ 418	− 2
Point 3	D. L.	+ 18,462	− 1,439	8,670	+ 17,465	2,982	+ 20,447	+ 104	− 33	+ 71	+ 137	+ 71	+ 71	+ 137	+ 137
sin ϕ=.344.	+ C. L. L.	+	+		+		+	+	+						
cos ϕ=.946.	+ U. L. L.	+ 4,060	+ 2,304	958	+ 3,841	330	+ 4,171	+ 21	+ 53	+ 74	− 32	+ 74			− 32
Area=196.	− C. L. L.	+	−		+		+	+	−						
	− U L. L.	+ 3,850	− 3,559	2,042	+ 3,642	702	+ 4,344	+ 22	− 82	− 60	+ 104		− 60	+ 104	
$\frac{h}{2I}$=.02303.	+ T.	+ 2,762	+ 6,480	0	+ 2,613	0	+ 2,613	+ 13	+ 149	+ 162	− 136	+ 162			− 136
	− T.	− 3,682	− 8,638	0	− 3,483	0	− 3,483	− 18	− 199	− 217	+ 181		− 217	+ 181	
									Total excluding temperature			+ 145		+ 241	
											Total	+ 307	− 206	+ 422	− 31
Point 8	D. L.	+ 18,462	+ 198	2,100	+ 18,388	216	+ 18,604	+ 120	+ 7	+ 127	+ 113	+ 127	+ 127	+ 113	+ 113
sin ϕ=.103.	+ C. L. L.	+	+		+		+	+	+						
cos ϕ=.996.	+ U. L. L.	+ 3,955	+ 2,973	534	+ 3,939	55	+ 3,994	+ 26	+ 111	+ 137	− 85	+ 137			− 85
Area=155.	− C. L. L.	+	−		+		+	+	−						
	− U. L. L.	+ 3,955	− 2,523	591	+ 3,939	61	+ 4,000	+ 26	− 94	− 68	+ 120		− 68	+ 120	
$\frac{h}{2I}$=.03728.	+ T.	+ 2,762	− 2,745	0	+ 2,751	0	+ 2,751	+ 18	− 102	− 84	+ 120		− 84	+ 120	
	− T.	− 3,682	+ 3,660	0	− 3,667	0	− 3,667	− 24	+ 136	+ 112	− 160	+ 112			− 160
									Total excluding temperature			+ 264		+ 233	
											Total	+ 376	− 25	+ 353	− 132
Point 10½	D. L.	+ 18,462	− 388	0	+ 18,462	0	+ 18,462	+ 121	− 15	+ 106	+ 136	+ 106	+ 106	+ 136	+ 136
sin ϕ=0.	+ C. L. L.		+		+		+	+	+						
cos ϕ=1.00.	+ U. L. L.	+ 4,666	+ 2,322	0	+ 4,666	0	+ 4,666	+ 31	+ 91	+ 122	− 60	+ 122			− 60
Area=152.	− C. L. L.	+	−		+		+	+	−						
	− U. L. L.	+ 3,245	− 1,344		+ 3,245	0	+ 3,245	+ 21	− 52	− 31	+ 73		− 31	+ 73	
$\frac{h}{2I}$=.03901.	+ T.	+ 2,762	− 3,470		+ 2,762	0	+ 2,762	+ 18	− 135	− 117	+ 153		− 117	+ 153	
	− T.	− 3,682	+ 4,985		− 3,682	0	− 3,682	− 24	+ 194	+ 170	− 218	+ 170			− 218
									Total excluding temperature			+ 228		+ 209	
											Total	+ 398	− 42	+ 362	− 142

$N = H \cos \phi + V \sin \phi$ $f_c = \frac{N}{A} \pm M\frac{h}{2I}$

columns 77, 78, and 79. It should be remembered that the V_o given in the sum of column 70 is the dead-load vertical shear at point 0, and to get the vertical shear at any other point the loads between point 0 and the point under consideration must be subtracted from V_o.

We will consider next the live-load thrusts, moments, and shears. A uniform live load of 125 pounds per square foot has been assumed, and since we are considering a strip of the arch ring 1 foot wide, the live load at one point is $125dx$, or 375 pounds. We wish to find the maximum stress at each of the points, 0, 3, 8, and 10½ and the stress is a function of the moment, thrust, and shear. It is not necessarily maximum at the same time that the moment is, but it is so nearly so that this is assumed to be true. Inspecting column 64 it is seen that the maximum positive moment at point 0 is produced by placing a live load at all of the points which give a positive moment, and no load should be placed at the points which give a negative moment. Thus the maximum positive live-load moment at point 0 is found by multiplying the sum of all of the positive quantities in column 64 by 375 pounds as is done in the table following.

The horizontal thrust which occurs at the same time as this maximum moment is found by adding the quantities in column 62 for the points which give a positive moment and multiplying the sum by 375. The same procedure is followed for the vertical shear, using the quantities in column 63. The maximum negative moment and the thrust and shear which occur at the same time are found by placing the loads at all of the points which give negative moments. The live-load moments, thrusts, and shears at the other points are found in the same way, but it should be remembered that the values of V_o in column 63 are the values of the reaction at the left support, and if there are any loads to the left of the point under consideration the shear is found by adding the proper quantities in the V_o-column and then subtracting 1.00 for each load to the left. Then the quantity thus obtained is multiplied by the live load per load point as shown in the following computations:

MOMENTS, THRUSTS, AND SHEARS DUE TO LIVE LOAD

Live load = 125 pounds per square foot
= 125×3 = 375 pounds per load point

Point 0:

+M = 375×34.822 =	+13,058
H = 375×14.890 =	5,584
V = 375× 2.759 =	1,035
−M = 375×30.129 =	−11,298
H = 375× 6.204 =	2,326
V = 375× 7.241 =	2,715

Point 3:

+M = 375× 6.143 =	+2,304
H = 375×10.828 =	4,060
V = 375×(4.555−2) =	958
−M = 375× 9.490 =	−3,559
H = 375×10.266 =	3,850
V = 375× 5.445 =	2,042

Point 8:

+M = 375× 7.929 =	+2,973
H = 375×10.547 =	3,955
V = 375×(8.423−7) =	534
−M = 375× 6.728 =	−2,523
H = 375×10.547 =	3,955
V = 375× 1.577 =	591

Point 10½:

$+M=375\times 6.192=$	$+2,322$
$H=375\times 12.442=$	4,666
$V=375\times(3-3)=$	0
$-M=375\times 3.585=$	$-1,344$
$H=375\times 8.652=$	3,245
$V=375\times(7-7)=$	0

The horizontal thrust due to a change in temperature is computed from the formula $H_t=\frac{20\ etE_c}{C}$. If E_c be taken as 2,000,000 pounds per square inch or 144 x 2,000,000 pounds per square foot and e be taken as 0.000006, the formula becomes $H_t=\frac{34560}{C}t$. H_t is constant for all points of the arch ring, and its value is calculated and recorded in column 77. M_t is computed from the formula $M_t=-H_t\left(y-\frac{\Sigma\, y\,\Delta}{\Sigma\Delta}\right)$. Values of $\left(y-\frac{\Sigma\, y\,\Delta}{\Sigma\Delta}\right)$ are found in column 23 except for point 0 and the value for this point can be quickly computed. The value of V_t is zero in every case.

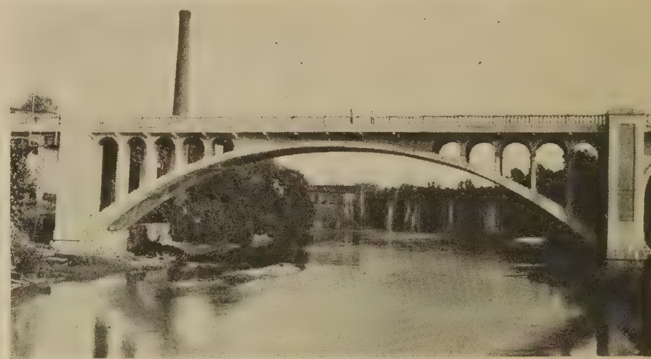
$H\cos\phi$ and $V\sin\phi$ are computed and tabulated in columns 80 and 81, respectively. N in column 82 is computed by adding the corresponding values in columns 80 and 81. Column 83 is computed by dividing N in column 82 by A in column 75. $M\frac{h}{2I}$ in column 84 is computed by multiplying M in column 78 by $\frac{h}{2I}$. The stress in the concrete at the extrados $\left(\frac{N}{A}+M\frac{h}{2I}\right)$ is computed for column 85 by adding the corresponding values in columns 83 and 84. The stress in the concrete at the intrados is computed in column 86 by subtracting column 84 from column 83, observing the correct algebraic sign.

Columns 85 and 86 give the stresses due to various conditions of loading and temperature, but those conditions do not all obtain at one time. To find the maximum stress which may obtain, columns 87 to 90 are used, as indicated in the headings. For each case the maximum stress is caused by a combination of the dead load, one position of the live load, and one condition of the temperature. These stresses are tabulated in columns 87 to 90, inclusive, and the algebraic sum taken. The correct algebraic sign must be observed in all cases.

These maximum stresses have been computed upon the assumption that the arch ring is uniformly elastic and acts in accordance with Hook's law under all conditions—that is, we have assumed that the concrete will withstand the tensile stresses as well as the compressive. If the computed tensile stresses do not exceed the allowable tensile stress for concrete, the above assumption is practically true, but if we find that the tension in the concrete exceeds the allowable stress at any point the assumption is not true and we must assume that the concrete cracks at that point and all of the tension is taken by the steel reinforcement. The stresses in the steel and concrete at that point should then be computed by another method.

This may be done by dividing the maximum moment by the maximum thrust (columns 78 and 82) to find the eccentricity of the thrust and then computing the stresses by the theory of bending and direct stress in reinforced concrete for which formulas and diagrams may be found in any textbook on reinforced concrete.

Stresses in steel and concrete.—Columns 87 to 90, inclusive, give the stresses in the concrete computed upon the assumption that the concrete will withstand the stress to which it is subjected. It is found that the tension in the concrete at the abutment under certain conditions is 284 pounds per square inch. Under these circumstances the concrete will crack and all of the tension must be taken by the steel.



A Two-Ribbed Open-Spandrel Arch Designed by the Method Described Here but with Some Variation on Account of the Position of Load Points as Fixed by the Location of Columns

When the tensile strength of the concrete is exceeded, the stresses in the steel and concrete may be found from the diagram on page 405 of Hool and Johnson's "Concrete Engineers' Handbook." It will be noted that in this case there is reinforcement on the tension side only. The nomenclature used in the following computations, if not previously explained, is found on page 403 of Hool and Johnson.

At point 0:

	M	N
D. L.=	$-16,639$	$+23,142$
L. L.=	$-11,298$	$+3,437$
T.=	$-21,562$	$-3,086$
	$-49,499$	$+23,493$

$u=\frac{-49,499}{+23,493}=-2.11$ ft.

$e'=u+\left(\frac{h}{2}-d'\right)=2.11+0.98=3.09$ ft. $=37.1$ in.

$d=h-d'=2.30-0.17=2.13$ ft. $=25.6$ in.

$\frac{e'}{d}=\frac{37.1}{25.6}=1.45.$

Area of steel = 1.125 sq. in. (see fig. 5):

$p=\frac{1.125}{144\times 2.13}=0.0037$ per cent steel.

$K=\frac{Ne'}{bd^2}=\frac{23,493\times 37.1}{12\times(25.6)^2}=111.$

From diagram (Hool and Johnson) $f_c=630$ and $f_s=14,000$.

Therefore the arch is satisfactory at this point.

At point 3 the worst conditions of loading and temperature give 206 pounds tension in the concrete, which will cause the concrete to crack.

At point 3:

	M	N
D. L.=	$-1,439$	$+20,446$
L. L.=	$-3,559$	$+4,344$
T.=	$-8,638$	$-3,483$
	$-13,636$	$+21,308$

$u=\frac{-13,636}{+21,308}=0.64$ ft.

$e'=0.64+0.48=1.12$ ft. $=13.4$ in.

$d=13.6$ in.

$\frac{e'}{d}=\frac{13.4}{13.6}=0.985.$

$p=\frac{0.5625}{144\times 1.13}=0.00346$ per cent steel.

$K=\frac{Ne'}{bd^2}=\frac{21,305\times 13.4}{12\times(13.6)^2}=128.$

From diagram, $f_c=575$ and $f_s=7,500$.

At point 8:

	M	N
D. L.=	+198	+18, 604
L. L.=	+2, 973	+3, 994
T.=	+3, 660	−3, 667
	+6, 831	+18, 931

$$u=\frac{+6{,}831}{+18{,}931}=0.361$$

$e'=0.361+0.34=0.701$ ft. $=8.4$ in.

$d=10.2$ in.

$$\frac{e'}{d}=\frac{8.4}{10.2}=0.824$$

$$p=\frac{0.5625}{12\times 10.2}=0.0046.$$

$$K=\frac{Ne'}{bd^2}=\frac{18{,}931\times 8.4}{12\times(10.2)^2}=127.$$

From diagram, $f_c=460$ and $f_s=2{,}600$.

At point $10\frac{1}{2}$:

	M	N
D. L.=	−388	+18, 462
L. L.=	+2, 322	+4, 666
T.=	+4, 985	−3, 682
	+6, 919	+19, 446

$$u=\frac{+6{,}919}{+19{,}446}=+0.356 \text{ ft.}$$

$e'=0.356+0.333=0.689$ ft. $=8.3$ in.

$d=10.0$ in.

$$\frac{e'}{d}=\frac{8.3}{10}=0.83.$$

$$p=\frac{0.5625}{12\times 10}=0.0047 \text{ per cent steel.}$$

$$K=\frac{Ne'}{bd^2}=\frac{19{,}446\times 8.3}{12\times(10)^2}=134.$$

From diagram, $f_c=490$ and $f_s=2{,}900$.

Foundation pressure.—The maximum pressure on the foundation at the back of the abutment will be caused by the dead load, the live load which causes positive moment at the abutment, and a rise of temperature. The live load which causes a negative moment at the abutment may in some cases increase the foundation pressure, but usually it does not. It will increase the thrust, but on account of the moment being of opposite sign, it decreases the eccentricity which decreases the maximum pressure at the back of the abutment.

	M	H	V
D. L.=	−16, 639	+18, 462	14, 000
L. L.=	+13, 058	+5, 584	1, 035
T.=	+16, 174	+2, 762	0
	+12, 593	+26, 808	15, 035

$$u=\frac{+12{,}593}{+26{,}808}=+0.47 \text{ ft.}$$

The thrust at the abutment (point 0) may be obtained by plotting H and V on the drawing and scaling the thrust, which is found to be 30,700 pounds and is applied 0.47 feet above point 0.

The weight of the abutment and the fill above the abutment is 18,650 pounds, and the center of gravity of these loads is 4.9 feet back of the face of the abutment. This weight is combined graphically with the thrust from the arch, and we obtain a total pressure of 43,000 pounds applied 1.21 feet from the back of the abutment. The vertical component of this pressure is 33,685 pounds and the horizontal component is 26,808 pounds.

The maximum vertical pressure on the foundation is therefore $\frac{33{,}685\times 2}{3\times 1.21}=18{,}560$ per square foot, or 9.3 tons per square foot. The horizontal pressure of 26,808 pounds must be taken by friction on the foundation and pressure against the vertical rock back of the abutment.

PART 2.—ANALYSIS OF UNSYMMETRICAL ARCHES

PART 1 of this article, published in the preceding issue of PUBLIC ROADS, presented the derivation of formulas for arch design and the calculations for the design of a symmetrical arch using the forms which have been developed to lessen the labor and chances of error in arch calculations. It will be assumed that the reader is familiar with the preceding article, and an example will be worked out for an unsymmetrical arch, explaining only such steps as have not already been discussed.

FORMULAS FOR UNSYMMETRICAL ARCHES

The following formulas for unsymmetrical arches were derived. They are applicable to either symmetrical or unsymmetrical arches, but for symmetrical arches it is much easier to use them in the simplified form previously explained.

$$V_o=\frac{\frac{1}{2}\sum_a^l(z-k)\Delta\left(z-\frac{\Sigma z\Delta}{\Sigma\Delta}\right)-\frac{\boldsymbol{G}}{\boldsymbol{C}}\times\frac{1}{2}\sum_a^l(z-k)\Delta\left(y-\frac{\Sigma y\Delta}{\Sigma\Delta}\right)}{\boldsymbol{F}-\frac{\boldsymbol{BG}}{\boldsymbol{C}}}$$

$$H_o=\frac{V_o\boldsymbol{B}-\frac{1}{2}\sum_a^l(z-k)\Delta\left(y-\frac{\Sigma y\Delta}{\Sigma\Delta}\right)}{\boldsymbol{C}}$$

$$M_o=\frac{dx}{\Sigma\Delta}\frac{1}{2}\sum_a^l(z-k)\Delta+H_o\frac{\Sigma y\Delta}{\Sigma\Delta}-V_o\frac{dx}{2}\frac{\Sigma z\Delta}{\Sigma\Delta}$$

$$M_x=M_o+[V_oz-(z-k)]\frac{dx}{2}-H_oy$$

$$V_t=\frac{\frac{2r}{(dx)^2}+20\frac{\boldsymbol{G}}{\boldsymbol{C}}}{\boldsymbol{F}-\frac{\boldsymbol{BG}}{\boldsymbol{C}}}etE$$

$$H_t=\frac{V_t\boldsymbol{B}+20\,etE}{\boldsymbol{C}}$$

$$M_t=-H_t\left(y-\frac{\Sigma y\Delta}{\Sigma\Delta}\right)+V_t\frac{dx}{2}\left(z-\frac{\Sigma z\Delta}{\Sigma\Delta}\right)$$

$$N_x=H\cos\phi+V_x\sin\phi$$

$$f_c=\frac{N}{A}\pm M\frac{h}{2I}$$

$$\boldsymbol{B}=\frac{1}{2}\Sigma z\Delta\left(y-\frac{\Sigma y\Delta}{\Sigma\Delta}\right)$$

$$\boldsymbol{C}=\frac{1}{dx}\Sigma y\Delta\left(y-\frac{\Sigma y\Delta}{\Sigma\Delta}\right)+\Sigma\frac{\cos\phi}{A}$$

$$\boldsymbol{F}=\frac{1}{2}\Sigma z\Delta\left(z-\frac{\Sigma z\Delta}{\Sigma\Delta}\right)$$

$$\boldsymbol{G}=\frac{1}{dx}\Sigma z\Delta\left(y-\frac{\Sigma y\Delta}{\Sigma\Delta}\right)$$

CALCULATIONS FOR AN UNSYMMETRICAL ARCH

An unsymmetrical arch of the dimensions shown in Figure 1 is to be designed to carry a live load of 125 pounds per square foot and to withstand a rise of temperature of 30° F. or a fall of 40°.

The general method of procedure is identical with that described for symmetrical arches. The dimensions of the arch ring and reinforcement may be arrived at by any of the methods in general use, and a drawing similar to Figure 1 is prepared on a scale of 1 inch to 3 feet. For unsymmetrical arches it is necessary to draw the entire arch ring.

Computations are entered in 12 tables, which can be conveniently arranged on six sheets of letter-size paper grouped as presented here. Bold-face type is used to indicate the results of calculations, and plus and minus signs which are the same for all arches and may be printed on the blank forms. Since the arch is unsymmetrical, the tables must be filled out for the entire arch in every case. As in the preceding example for a symmetrical arch, coefficients will be determined for a load of unity at the various points, and these coefficients will be applied to the dead and live load to determine the stresses.

Table 1. Computations for Δ.—This table is filled out in the same way as Table 1 for the symmetrical arch and requires no additional explanation.

Table 2. Computations for $\frac{1}{2}\sum_a^l(z-k)\Delta\left(y-\frac{\Sigma y\Delta}{\Sigma\Delta}\right)$.—The value of this expression, which appears in the formula for V_o, was not found for the symmetrical arch, since it is multiplied by $\boldsymbol{G}$, which in the case of the symmetrical arch is zero. The value of y for each of the 20 points is scaled and recorded in column 12. The value of $y\Delta$, column 13, is the product of the corresponding values of y and Δ in columns 12 and 11. The sum of column 13 should be divided by the sum of column 11 and the value of $\frac{\Sigma y\Delta}{\Sigma\Delta}$ is recorded in the place provided on the same sheet with Tables 1 and 2.

Column 14 is computed by subtracting $\frac{\Sigma y\Delta}{\Sigma\Delta}$ from each value of y. Column 15 is computed by multiplying each term in column 14 by the corresponding term in column 11. If the work in columns 13, 14, and 15 is correct, the sum of column 15 will be zero.

Columns 16 and 17 are computed in exactly the same manner as explained for columns 36 and 37 of Table 4 for symmetrical arches. The figure opposite point 20 in column 16 is always zero. To this add the figure opposite point 20 in column 15 with its proper algebraic sign and write the result in the next space in column 16. To this add algebraically the next figure in column 15 which is opposite point 19, and set down the sum in column 16 opposite point 18. Then add the next figure in column 15 and continue this process to the top of the page. Since the sum of column 15 is zero, the top figure in column 16, opposite point 1, will be numerically equal to the top figure in column 15.

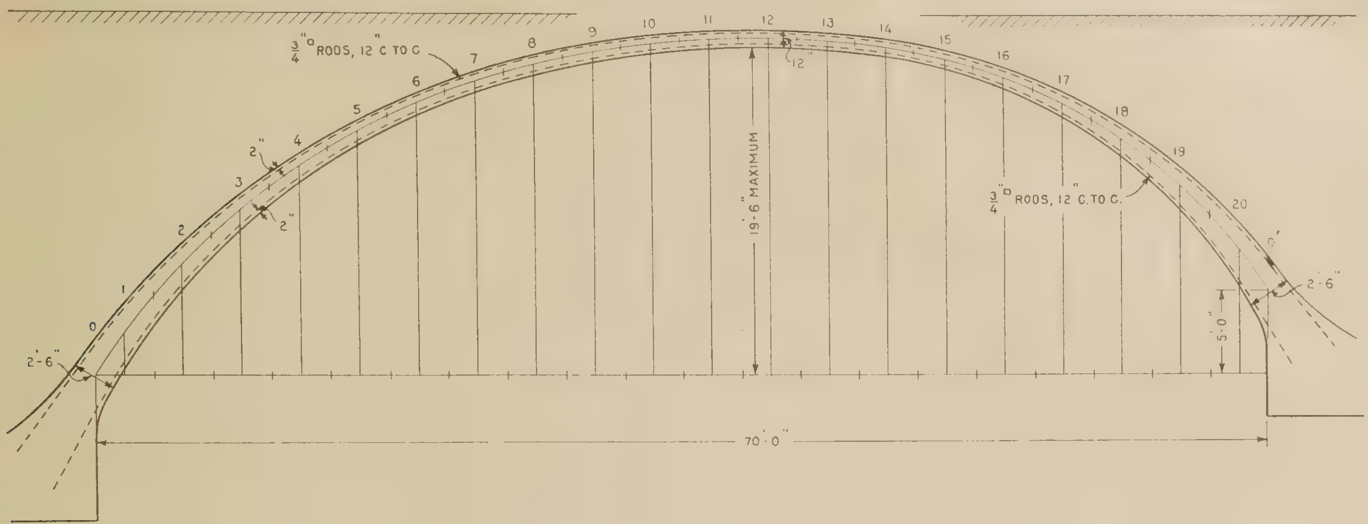


FIG. 1.—SKETCH OF ARCH RING SHOWING PRINCIPAL DIMENSIONS AND LOAD POINTS

TABLE 1.—*Computations for* Δ

TABLE 2.—*Computations for* $\frac{1}{2}\Sigma(z-k)\Delta\left(y-\frac{\Sigma y\Delta}{\Sigma\Delta}\right)$

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
Points	h	h^3	I_c	$\frac{h}{2}$	$\left(\frac{h}{2}-d'\right)$	$\left(\frac{h}{2}-d'\right)^2$	I_s	$I=I_c+I_s$	ds	$\Delta=\frac{ds}{I}$	y	$y\Delta$	$y-\frac{\Sigma y\Delta}{\Sigma\Delta}$	$\Delta\left(y-\frac{\Sigma y\Delta}{\Sigma\Delta}\right)$	$\sum_a^l\Delta\left(y-\frac{\Sigma y\Delta}{\Sigma\Delta}\right)$	$\frac{1}{2}\sum_a^l(z-k)\Delta\times\left(y-\frac{\Sigma y\Delta}{\Sigma\Delta}\right)$
0	2.50	15.625	1.302	1.25	1.08	1.166	0.136	1.438	—	—	0.00	—	—	—	—	—
1	2.32	12.487	1.041	1.16	.99	.980	.115	1.156	6.05	5.23	2.55	13.3365	− 14.950856	− 78.1930	+ 78.1932	+ 2347.96
2	1.92	7.078	.590	.96	.79	.624	.073	.663	5.30	7.99	7.10	56.7290	− 10.400856	− 83.1028	+ 161.2960	+ 2269.77
3	1.60	4.096	.341	.80	.63	.397	.046	.387	4.65	12.02	10.55	126.8110	− 6.950856	− 83.5493	+ 244.8453	+ 2108.48
4	1.34	2.406	.200	.67	.50	.250	.029	.229	4.10	17.90	13.06	233.7740	− 4.440856	− 79.4913	+ 324.3366	+ 1863.63
5	1.20	1.728	.144	.60	.43	.185	.022	.166	3.93	23.67	15.06	356.4702	− 2.440856	− 57.7750	+ 382.1116	+ 1539.29
6	1.10	1.331	.111	.55	.38	.144	.017	.128	3.75	29.30	16.58	485.7940	− .920856	− 26.9811	+ 409.0927	+ 1157.18
7	1.08	1.260	.105	.54	.37	.137	.016	.121	3.65	30.17	17.68	533.4056	+ .179144	+ 5.4048	+ 403.6879	+ 748.09
8	1.06	1.191	.099	.53	.36	.130	.015	.114	3.60	31.58	18.60	587.3880	+ 1.099144	+ 34.7110	+ 368.9769	+ 344.40
9	1.04	1.125	.094	.52	.35	.122	.014	.108	3.55	32.87	19.26	633.0762	+ 1.759144	+ 57.8231	+ 311.1538	− 24.58
10	1.02	1.061	.088	.51	.34	.116	.014	.102	3.50	34.31	19.72	676.5932	+ 2.219144	+ 76.1388	+ 235.0150	− 335.73
11	1.00	1.000	.083	.50	.33	.109	.013	.096	3.50	36.46	19.92	726.2832	+ 2.419144	+ 88.2020	+ 146.8130	− 570.74
12	1.00	1.000	.083	.50	.33	.109	.013	.096	3.50	36.46	20.00	729.2000	+ 2.499144	+ 91.1188	+ 55.6942	− 717.56
13	1.00	1.000	.083	.50	.33	.109	.013	.096	3.51	36.56	19.95	729.3720	+ 2.499144	+ 89.5407	− 33.8465	− 773.25
14	1.02	1.061	.088	.51	.34	.116	.014	.102	3.54	34.71	19.62	681.0102	+ 2.119144	+ 73.5555	− 107.4020	− 739.40
15	1.06	1.091	.099	.53	.36	.130	.015	.114	3.60	31.58	19.00	600.0200	+ 1.499144	+ 47.3430	− 154.7450	− 632.00
16	1.10	1.331	.111	.55	.38	.144	.017	.128	3.70	28.91	18.05	521.8255	+ .549144	+ 15.8757	− 170.6207	− 477.26
17	1.20	1.728	.144	.60	.43	.185	.022	.166	3.88	23.37	16.60	387.9420	− .900856	− 21.0530	− 149.5677	− 306.64
18	1.40	2.744	.229	.70	.53	.281	.033	.262	4.25	16.22	14.60	236.8120	− 2.900856	− 47.0519	− 102.5158	− 157.07
19	1.84	6.230	.519	.92	.75	.562	.066	.585	4.88	8.34	11.75	97.9950	− 5.750856	− 47.9621	− 54.5537	− 54.55
20	2.26	11.543	.962	1.13	.96	.922	.108	1.070	5.90	5.51	7.60	41.8760	− 9.900856	− 54.5537	0.0000	0.00
0′	2.50	15.625	1.302	1.25	1.08	1.166	.136	1.438	—	—	5.00	—	—	—	—	—
										ΣΔ 483.16		$\Sigma y\Delta=$ 8455.7136		0.0000		+ 7,590.02

Span $l=70.00$ feet. $dx=3.50$ feet. $\frac{\Sigma y\Delta}{\Sigma\Delta}=17.500856$ $I_c=\frac{h^3}{12}$ $I_s=\left(\frac{h}{2}-d'\right)^2 nA_s$

The figure in column 17 opposite point 20 is zero. In the next space opposite point 19 set down the figure taken from column 16 opposite point 19 with its proper algebraic sign. To this add algebraically the figure taken from column 16 opposite point 18 and set down the result in column 17 opposite point 18. Continue this summation to the top of the page. Column 17 will then contain the values $\frac{1}{2}\sum_a^l(z-k)\Delta\left(y-\frac{\Sigma y\Delta}{\Sigma\Delta}\right)$ for a load of unity placed successively at each load point.

Table 3. Computations for $\frac{1}{2}\sum_a^l(z-k)\Delta\left(z-\frac{\Sigma z\Delta}{\Sigma\Delta}\right)$.—The values of z which are always the same when the arch ring is divided into 20 parts, are permanently printed in column 19. Columns 20, 21, and 22 are computed as indicated at the heads of the columns. After column 20 is computed the value of $\frac{\Sigma z\Delta}{\Sigma\Delta}$ is determined and the result set down in the space provided on the sheet containing Tables 3 and 4. If the arch were symmetrical $\frac{\Sigma z\Delta}{\Sigma\Delta}$ would equal 20, and when unsymmetrical it should not differ from 20 a great amount unless the arch is very far from being symmetrical.

Column 23 is computed from column 22 in the same way that column 16 was computed from column 15. The sum of column 22 should be zero if the numerical work is correct and the top figure in column 23 opposite point 1 should be numerically equal to the top figure in column 22. Column 24 is computed from column 23 in the same way that column 17 was computed from column 16. Column 24 will contain the values of $\frac{1}{2}\sum_a^l(z-k)\Delta\left(z-\frac{\Sigma z\Delta}{\Sigma\Delta}\right)$ for a load of unity placed successively at each load point.

Table 4.—Computations for ***B, C, F, and G.***—These terms are constant for any particular arch ring, since

TABLE 3.—*Computations for* $\frac{1}{2}\sum_a^l (z-k)\Delta\left(z-\frac{\Sigma z\Delta}{\Sigma\Delta}\right)$ TABLE 4.—*Computations for* ***B, C, F,*** *and* ***G***

18	19	20	21	22	23	24	25	26	27	28	29
Points	z	$z\Delta$	$z-\frac{\Sigma z\Delta}{\Sigma\Delta}$	$\Delta\left(z-\frac{\Sigma z\Delta}{\Sigma\Delta}\right)$	$\sum_a^l \Delta\left(z-\frac{\Sigma z\Delta}{\Sigma\Delta}\right)$	$\frac{1}{2}\sum_a^l (z-k)\Delta\times\left(z-\frac{\Sigma z\Delta}{\Sigma\Delta}\right)$	$z\Delta\left(y-\frac{\Sigma y\Delta}{\Sigma\Delta}\right)$	$y\Delta\left(y-\frac{\Sigma y\Delta}{\Sigma\Delta}\right)$	$\cos\phi$	$\frac{\cos\phi}{A}$	$z\Delta\left(z-\frac{\Sigma z\Delta}{\Sigma\Delta}\right)$
0	0	—	—	—	—	—	—	—	0.549	—	—
1	1	5.23	− 19.606714	− 102.5431	+ 102.5432	19,309.05	− 78.19	− 199.39	.575	.248	− 102.54
2	3	23.97	− 17.606714	− 140.6776	+ 243.2208	19,206.51	− 249.31	− 590.03	.666	.347	− 422.03
3	5	60.10	− 15.606714	− 187.5927	+ 430.8135	18,963.29	− 417.75	− 881.44	.764	.477	− 937.96
4	7	125.30	− 13.606714	− 243.5602	+ 674.3737	18,532.48	− 556.44	− 1,038.16	.844	.630	− 1,704.92
5	9	213.03	− 11.606714	− 274.7309	+ 949.1046	17,858.10	− 519.98	− 870.09	.896	.746	− 2,472.58
6	11	322.30	− 9.606714	− 281.4767	+ 1230.5813	16,909.00	− 296.79	− 447.35	.940	.854	− 3,096.24
7	13	392.21	− 7.606714	− 229.4946	+ 1460.0759	15,678.42	+ 70.26	+ 95.56	.962	.890	− 2,983.43
8	15	473.70	− 5.606714	− 177.0600	+ 1637.1359	14,218.34	+ 520.67	+ 645.62	.975	.920	− 2,655.90
9	17	558.79	− 3.606714	− 118.5527	+ 1755.6886	12,581.21	+ 982.99	+ 1,113.67	.987	.948	− 2,015.40
10	19	651.89	− 1.606714	− 55.1246	+ 1810.8150	10,825.52	+ 1,446.64	+ 1,501.46	.996	.976	− 1,047.40
11	21	765.66	+ 0.393286	+ 14.3392	+ 1796.4758	9,014.70	+ 1,852.24	+ 1,756.98	.999	.999	+ 301.12
12	23	838.58	+ 2.393286	+ 87.2592	+ 1709.2166	7,218.23	+ 2,095.73	+ 1,822.38	1.000	1.000	+ 2,006.96
13	25	914.00	+ 4.393286	+ 160.6185	+ 1548.5981	5,509.01	+ 2,238.52	+ 1,786.34	.998	.998	+ 4,015.46
14	27	937.17	+ 6.393286	+ 221.9110	+ 1326.6871	3,960.41	+ 1,986.00	+ 1,443.16	.990	.970	+ 5,991.60
15	29	915.82	+ 8,393286	+ 265.0600	+ 1061.6271	2,633.72	+ 1,372.95	+ 899.52	975	.920	+ 7,686.74
16	31	896.21	+ 10.393286	+ 300.4699	+ 761.1572	1,572.10	+ 492.15	+ 286.56	.948	.862	+ 9 314.57
17	33	771.21	+ 12 393286	+ 289 6311	+ 471.5261	810.94	− 694 75	− 349.48	.903	.752	+ 9 557.83
18	35	567.70	+ 14.393286	+ 233 4591	+ 238.0670	339 41	− 1,646 82	− 686 96	.812	580	+ 8.171.07
19	37	308.58	+ 16 393286	+ 136.7200	+ 101.3470	101 35	− 1.774 60	− 563.55	.718	.390	+ 5.058.64
20	39	214.89	+ 18.393286	+ 101.3470	0.0000	0.00	− 2,127.59	− 414.61	.605	.268	+ 3,952.53
0′	40	—	—	—	—	—	—	—	.558	—	—
		$\Sigma z\Delta$=9956.34		Σ=0.0000			Σ=+4,695.93	+ 5,310.19		Σ=14.775	+ 38,618.12

$$\frac{\Sigma z\Delta}{\Sigma\Delta}=20.606{,}714$$

$$C=\frac{1}{dx}\sum_o^l y\Delta\left(y-\frac{\Sigma y\Delta}{\Sigma\Delta}\right)+\sum_o^l\frac{\cos\phi}{A}=1{,}531.97$$

$$\frac{G}{C}=.875797$$

$$F=\frac{1}{2}\sum_o^l z\Delta\left(z-\frac{\Sigma z\Delta}{\Sigma\Delta}\right)=19{,}309.06$$

$$\frac{BG}{C}=2{,}056.34$$

$$B=\frac{1}{2}\sum_o^l z\Delta\left(y-\frac{\Sigma y\Delta}{\Sigma\Delta}\right)=\frac{1}{2}\sum^l \text{Col. } 25=2{,}347.965$$

$$G=\frac{1}{dx}\sum_o^l z\Delta\left(y-\frac{\Sigma y\Delta}{\Sigma\Delta}\right)=1{,}341.6943$$

$$F-\frac{BG}{C}=17{,}252.72$$

$$\frac{1}{C}=0.000{,}652{,}75$$

$$\frac{1}{F-\frac{BG}{C}}=0.000{,}057{,}961{,}9$$

they are independent of the loading. The values in column 25 are computed by multiplying each term in column 15 by the corresponding value of z in column 19. ***B*** is equal to one-half the sum of column 25 and ***G*** is equal to $\frac{1}{dx}$ times the sum of column 25. Their values should be set down in the places provided on the sheet containing Tables 3 and 4.

Column 26 is computed by multiplying each term in column 15 by the corresponding value of y in column 12. Cos ϕ (see fig. 3, Pt. I) in column 27 might be computed from the formula $\cos\phi=\frac{dx}{ds}$, but since $\cos\phi$ varies so little for small angles more accurate results will be obtained by computing $\sin\phi$ from dimensions scaled from the drawing and then taking the corresponding values of $\cos\phi$ from a table of trigonometric functions. This may be done on a supplementary sheet of paper. It is not necessary to find the values of ϕ as only $\cos\phi$ is wanted now, but the values of $\sin\phi$ should be kept because some of them will be wanted later.

Column 28 is computed by dividing each term in column 27 by A, the area of the arch ring section, which for practical purposes may be taken as equal to h (column 2).

C is equal to the sum of column 28 plus $\frac{1}{dx}$ times the sum of column 26. Its value should be set down in the space provided below Table 4.

Column 29 is computed by multiplying each term of column 22 by the corresponding value of z in column 19. ***F*** is equal to one-half the sum of column 29 and its value should be set down in the space provided below Table 4. Values of other expressions below Table 4 are found as indicated and require no particular explanation.

Table 5. Computations for V_o.—V_o is computed from the formula

$$V_o=\frac{\frac{1}{2}\sum_a^l (z-k)\Delta\left(z-\frac{\Sigma z\Delta}{\Sigma\Delta}\right)-\frac{G}{C}\frac{1}{2}\sum_a^l (z-k)\Delta\left(y-\frac{\Sigma y\Delta}{\Sigma\Delta}\right)}{F-B\frac{G}{C}}$$

Column 31 is copied from column 24. Column 32 is computed by multiplying each term in column 17 by $\frac{G}{C}$. Column 33 is computed by subtracting each term in column 32 from the corresponding term in column 31 and the numerical work may be checked by subtracting the sum of column 32 from the sum of column 31. Column 34 is computed by dividing each term in column 33 by $\left(F-B\frac{G}{C}\right)$. Column 34 now contains the values of V_o for a load of unity placed successively at each load point. The top figure in column 34, that is the value of V_o for a unit load at point 1, should equal unity, but allowance should be made for the inaccuracy due to dropping of decimals.

TABLE 5.—*Computations for V_o* | TABLE 6.—*Computations for H_o* | TABLE 7.—*Computations for M_o*

30	31	32	33	34	35	36	37	38	39	40	41	42	43	44
Points	$\frac{1}{2}\sum_a^l (z-k)\Delta \times \left(z-\frac{\Sigma z\Delta}{\Sigma\Delta}\right)$	$\frac{G}{C}\times$Col. 17	Col. 31−Col. 32	$V_o=\frac{\text{Col. 33}}{F-\frac{BG}{C}}$	V_oB	Col. 35−Col. 17	$H_o=\frac{\text{Col. 36}}{C}$	Δ	$\sum_a^l \Delta$	$\frac{1}{2}\sum_a^l (z-k)\Delta$	$\frac{dx}{\Sigma\Delta}\times$ Col. 40	$H_o \frac{\Sigma y\Delta}{\Sigma\Delta}\times$	$V_o\frac{dx}{2}\frac{\Sigma z\Delta}{\Sigma\Delta}$	$M_o=$ Col. 41+ Col. 42− Col. 43
0														
1	19,309.05	+2,056.34	17,252.71	1.0000	2,347.96	0.00	0.000	5.23	477.93	4,736.59	34.312	0.000	36.062	− 1.750
2	19,206.51	+1,987.86	17,218.65	.9980	2,343.27	73.50	.048	7.99	469.94	4,258.66	30.850	.840	35.990	− 4.300
3	18,963.29	+1,846.60	17,116.69	.9921	2,329.42	220.94	.144	12.02	457.92	3,788.72	27.445	2.520	35.777	− 5.812
4	18,532.48	+1,632.16	16,900.32	.9796	2,300.07	436.44	.285	17.90	440.02	3,330.80	24.128	4.988	35.326	− 6.210
5	17,858.10	+1,348.11	16,509.99	.9569	2,246.77	707.48	.462	23.67	416.35	2,890.78	20.941	8.085	34.507	− 5.481
6	16,909.00	+1,013.45	15,895.55	.9213	2,163.18	1,006.00	.657	29.30	387.05	2,474.43	17.925	11.498	33.224	− 3.801
7	15,678.42	+ 655.17	15,023.25	.8708	2,044.61	1,296.52	.846	30.17	356.88	2,087.38	15.121	14.806	31.403	− 1.476
8	14,218.34	+ 301.62	13,916.72	.8066	1,893.87	1,549.47	1.011	31.58	325.30	1,730.50	12.536	17.693	29.087	+ 1.142
9	12,581.21	− 21.53	12,602.74	.7305	1,715.19	1,739.77	1.136	32.87	292.43	1,405.20	10.179	19.881	26.343	+ 3.717
10	10,825.52	− 294.03	11,119.55	.6445	1,513.26	1,848.99	1.207	34.31	258.12	1,112.77	8.061	21.124	23.242	+ 5.943
11	9,014.70	− 499.85	9,514.55	.5515	1,294.90	1,865.64	1.218	36.46	221.66	854.65	6.191	21.316	19.888	+ 7.619
12	7,218.23	− 628.44	7,846.67	.4548	1,067.85	1,785.41	1.165	36.46	185.20	632.99	4.585	20.388	16.401	+ 8.572
13	5,509.01	− 677.21	6,186.22	.3586	841.98	1,615.23	1.054	36.56	148.64	447.79	3.244	18.446	12.932	+ 8.758
14	3,960.41	− 647.56	4,607.97	.2671	627.14	1,366.54	.892	34.71	113.93	299.15	2.167	15.611	9.632	+ 8.146
15	2,633.72	− 553.50	3,187.22	.1847	433.67	1,065.67	.696	31.58	82.35	185.22	1.342	12.181	6.661	+ 6.862
16	1,572.10	− 417.98	1,990.08	.1153	270.72	747.98	.488	28.91	53.44	102.87	.745	8.540	4.158	+ 5.127
17	810.94	− 268.55	1,079.49	.0626	146.98	453.62	.296	23.37	30.07	49.43	.358	5.180	2.257	+ 3.281
18	339.41	− 137.56	476.97	.0276	64.80	221.87	.145	16.22	13.85	19.36	.140	2.538	.995	+ 1.683
19	101.35	− 47.77	149.12	.0086	20.19	74.74	.049	8.34	5.51	5.51	.040	.857	.310	+ .587
20	0.00	0.00	0.00	.00	0.00	0.00	.00	5.51	0.00	0.00	0.000	0.000	0.000	0.000
0′														
	195,241.79	6,647.33	188.594.46	10.9311	25,665.83	18,075.81	11.799	------	--------	30,412.80	220.310	206.492	394.195	+ 32.607

$$V_o=\frac{\frac{1}{2}\sum_a (z-k)\Delta\left(z-\frac{\Sigma z\Delta}{\Sigma\Delta}\right)-\frac{G}{C}\frac{1}{2}\sum_a^l (z-k)\Delta\left(y-\frac{\Sigma y\Delta}{\Sigma\Delta}\right)}{F-\frac{BG}{C}}$$

$$M_o=\frac{dx}{\Sigma\Delta}\frac{1}{2}\sum_a^l (z-k)\Delta+H_o\frac{\Sigma y\Delta}{\Sigma\Delta}-V_o\frac{dx}{2}\frac{\Sigma z\Delta}{\Sigma\Delta}$$

$$H_o=\frac{V_oB-\frac{1}{2}\sum_a^l\left(z-k\right)\Delta\left(y-\frac{\Sigma y\Delta}{\Sigma\Delta}\right)}{C}$$

$$\frac{dx}{\Sigma\Delta}=0.007244 \qquad \frac{dx}{2}\frac{\Sigma z\Delta}{\Sigma\Delta}=36.06175$$

Table 6. Computations for H_o.—H_o is computed from the formula,

$$H_o=\frac{V_o\boldsymbol{B}-\frac{1}{2}\sum_a^l (z-k)\Delta\left(y-\frac{\Sigma y\Delta}{\Sigma\Delta}\right)}{\boldsymbol{C}}$$

V_ob is computed by multiplying each term in column 34 by the value of $\boldsymbol{B}$ and entered in column 35. The term $\frac{1}{2}\sum_a^l (z-k)\Delta\left(y-\frac{\Sigma y\Delta}{\Sigma\Delta}\right)$ has already been recorded in column 17, so the numerator of the equation for H_o is computed by subtracting each term of column 17 from the corresponding term of column 35. The numerical work is checked by subtracting the sum of column 17 from the sum of column 35. H_o is then computed by dividing each term in column 36 by $\boldsymbol{C}$. The top figure of column 35 should be equal to the top figure of column 17 and the top figure of column 36, should be zero.

Table 7.—Computations for M_o.—M_o is computed from the formula,

$$M_o=\frac{dx}{\Sigma\Delta}\frac{1}{2}\sum_a^l (z-k)\Delta+H_o\frac{\Sigma y\Delta}{\Sigma\Delta}-V_o\frac{dx}{2}\frac{\Sigma z\Delta}{\Sigma\Delta}$$

For convenience the values of Δ are copied in column 38 from column 11. Columns 39 and 40 are computed by summing up column 38 in the same way as previously explained for other similar columns. Columns 41, 42 and 43 are computed as indicated by the headings and no further explanation is necessary. M_o is computed in column 44 by adding the corresponding members of columns 41 and 42 and subtracting the corresponding member of column 43. M_o for a unit load at point 1 is always numerically equal to $\frac{dx}{2}$ and the numerical work in computing column 44 may be checked from the totals of columns 41, 42 and 43.

Tables 8, 9, and 10. Computations for moments at points 2, 11 and 0′.—Having determined the values of H_o, V_o, and M_o, other points must now be selected for detailed examination as to stress conditions. In this case points 2 and 11 have been selected and point 0′ should always be included. Values of H_o, V_o and M_o are copied in columns 46, 47 and 48.

The bending moment at any point is computed from the formula

$$M_x=M_o+m_x-H_oy.$$

For any particular point, z and y are constant and k depends on the position of the load. For point 2, z is equal to 3 and the value of y is taken from column 12 of Table 2.

Column 49 is computed by multiplying the values of V_o in column 47 by z. This is done only for the points to the left of the one for which the moment is being computed because the formula $m_x=[V_oz-(z-k)]\frac{dx}{2}$ is used only where z is greater than k. For the points to the right, m_x is found by the formula $m_x=V_oz\frac{dx}{2}$. Column 50 is computed by subtracting $(z-k)$ from the values of V_oz in column 49. For a unit load at point 1, $V_oz-(z-k)$ will be unity in all cases so its value is permanently printed in the tables. In column 51, M_2 is computed for points 1 and 2 by multiplying

TABLE 8.—*Computations for* M_2 TABLE 9.—*Computations for* M_{11} TABLE 10.—*Computations for* M_o'

45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63
Points	H_o	V_o	M_o	Point 2, $z=3$; $y=7.10$					Point 11, $z=21$; $y=19.92$					Point 0′, $z=40$; $y=5.00$				
				V_oz	$V_oz-(z-k)$	m_2	H_oy	M_2	V_oz	$V_oz-(z-k)$	m_{11}	H_oy	M_{11}	$40V_o$	$40V_o-(40-k)$	m_o'	H_oy	M_o'
0	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
1	0.000	1.0000	− 1.750	3.0000	1.0000	1.750	0.000	0.000	21.0000	1.0000	1.750	0.000	0.000	40.0000	+ 1.0000	+ 1.750	0.000	0.000
2	.048	.9980	− 4.300	2.9940	2.9940	5.240	.341	+ .599	20.9580	2.9580	5.176	.956	− .080	39.9200	+ 2.9200	+ 5.110	.240	+ .570
3	.144	.9921	− 5.812			5.209	1.022	−1.625	20.8341	4.8341	8.460	2.868	− .220	39.6840	+ 4.6840	+ 8.197	.720	+ 1.665
4	.285	.9796	− 6.210			5.143	2.024	−3.091	20.5716	6.5716	11.500	5.677	− .387	39.1840	+ 6.1840	+ 10.822	1.425	+ 3.187
5	.462	.9569	− 5.481			5.024	3.280	−3.737	20.0949	8.0949	14.166	9.203	− .518	38.2760	+ 7.2760	+ 12.733	2.310	+ 4.942
6	.657	.9213	− 3.801			4.837	4.665	−3.629	19.3473	9.3473	16.358	13.087	− .530	36.8520	+ 7.8520	+ 13.741	3.285	+ 6.655
7	.846	.8708	− 1.476			4.572	6.007	−2.911	18.2868	10.2868	18.002	16.852	− .326	34.8320	+ 7.8320	+ 13.706	4.230	+ 8.000
8	1.011	.8066	+ 1.142			4.235	7.178	−1.801	16.9386	10.9386	19.143	20.139	+ .146	32.2640	+ 7.2640	+ 12.712	5.055	+ 8.799
9	1.136	.7305	+ 3.717			3.835	8.066	− .514	15.3405	11.3405	19.846	22.629	+ .934	29.2200	+ 6.2200	+ 10.885	5.680	+ 8.922
10	1.207	.6445	+ 5.943			3.384	8.570	+ .757	13.5345	11.5345	20.185	24.043	+2.085	25.7800	+ 4.7800	+ 8.365	6.035	+ 8.273
11	1.218	.5515	+ 7.619			2.895	8.648	+1.866			20.268	24.263	+3.624	22.0600	+ 3.0600	+ 5.355	6.090	+ 6.884
12	1.165	.4548	+ 8.572			2.388	8.271	+2.689			16.714	23.207	+2.079	18.1920	+ 1.1920	+ 2.086	5.825	+ 4.833
13	1.054	.3586	+ 8.758			1.883	7.483	+3.158			13,179	20.996	+ .941	14.3440	− .6560	− 1.148	5.270	+ 2.340
14	.892	.2671	+ 8.146			1.402	6.333	+3.215			9.816	17.769	+ .193	10.6840	− 2.3160	− 4.053	4.460	− .367
15	.696	.1847	+ 6.862			.970	4.942	+2.890			6.788	13.864	− .214	7.3880	− 3.6120	− 6.321	3.480	− 2.939
16	.488	.1153	+ 5.127			.605	3.465	+2.267			4.237	9.721	− .357	4.6120	− 4.3880	− 7.679	2.440	− 4.992
17	.296	.0626	+ 3.281			.329	2.102	+1.508			2.301	5.896	− .314	2.5040	− 4.4960	− 7.868	1.480	− 6.067
18	.145	.0276	+ 1.683			.145	1.030	+ .798			1.014	2.888	− .191	1.1040	− 3.8960	− 6.818	.725	− 5.860
19	.049	.0086	+ .587			.045	.348	+ .284			.316	.976	− .073	.3440	− 2.6560	− 4.648	.245	− 4.306
20	0.000	0.0000	0.000	0.000	0.0000	0.000	0.000	0.000	0.000	0.0000	0.00	0.000	0.000	0.0000	− 1.0000	− 1.750	0.000	− 1.750
0′	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
	11.799	10.9311	+32.607			53.891	83.775	+2.723			209.219	235.034	+6.792			+ 65.177	58.995	+ 38.789

$$M=M_o+m_x-H_oy$$

$$m_x=\left[\begin{array}{r} z>k \\ V_oz-(z-k)\end{array}\right]\frac{dx}{2}$$

For check:

Σ Col. 48 + Σ Col. 51 − Σ Col. 52 = Σ Col. 53.
Σ Col. 48 + Σ Col. 56 − Σ Col. 57 = Σ Col. 58.
Σ Col. 48 + Σ Col. 61 − Σ Col. 62 = Σ Col. 63.

the figure in column 50 by $\frac{dx}{2}$. For the other points m_2 is computed by multiplying the value of V_o in column 47 by $z\frac{dx}{2}$.

H_oy is computed by multiplying the values of H_o in column 46 by the value of y for the point under consideration. M_2 in column 53 is computed by adding the corresponding terms in columns 48 and 51 and subtracting the corresponding term in column 52. The numerical work may be checked from the totals of those columns. M_x for a load at point 1 will be zero in all cases because, for a unit load at point 1, M_o is equal $-\frac{dx}{2}$ and m_x is equal to $+\frac{dx}{2}$ and H_o is equal to zero.

Tables 9 and 10 are compiled in exactly the same manner as Table 8. Since M_o' should always be computed the numerical value of z is used in the column headings.

Table 11. Computations for thrust, shears, and moments due to dead load.—This table is filled out in the same manner as Table 8 of part 1. The values of H_o, V_o, M_o, M_2, M_{11}, and M_o' are copied from columns 37, 34, 44, 53, 58, and 63 in columns 65 to 70. The weight of the dead load applied at each point is computed as shown below and tabulated in column 71. The weight of the concrete is assumed to be 150 pounds per cubic foot and the fill above the arch ring 110 pounds per cubic foot. The dead load applied at each point is therefore equal to $150\,hds+110\,h\,dx$, in which h is the depth of the fill above the arch ring at each point and may be scaled from the drawing. Values of h and ds are found in columns 2 and 10 of Table 1. After column 71 is filled out the table is completed by multiplying the coefficients for a load of unity in the preceding columns by the actual dead load in order to get the values in the remaining columns of the table.

Calculation of dead load

Point	$h\ (ds)$	$150\ h(ds)$	h_f	$110\ dx\ h_f$	Dead load
1	14.05	2,105	17.1	6,600	8,700
2	10.20	1,530	13.3	5,130	6,660
3	7.45	1,115	10.1	3,900	5,015
4	5.50	825	7.7	2,960	3,785
5	4.72	708	5.9	2,270	2,978
6	4.13	620	4.5	1,730	2,350
7	3.95	592	3.4	1,310	1,902
8	3.82	573	2.5	960	1,533
9	3.70	555	1.8	690	1,245
10	3.57	535	1.3	500	1,035
11	3.50	525	1.0	385	910
12	3.50	525	1.0	385	910
13	3.51	527	1.0	385	912
14	3.61	541	1.4	540	1,081
15	3.82	573	2.0	770	1,343
16	4.07	610	2.9	1,120	1,730
17	4.66	700	4.3	1,660	2,360
18	5.95	893	6.1	2,350	3,243
19	9.00	1,350	8.5	3,270	4,620
20	13.35	2,003	12.1	4,650	6,653

Table 12. Computations for maximum stresses.—This table is the same as Table 9 (pt. 1) for symmetrical arches and is filled out in the same way except for the differences in the formulas due to the lack of symmetry. Column 78 is filled out as explained for column 75 of the symmetrical arch forms. The dead load moments, thrusts, and shears are found in Table 11, and copied in the proper places in columns 80, 81, and 82.

The live load has been assumed as 125 pounds per square foot which gives 125 dx = 437.5 pounds per load point. As in the case of the symmetrical arch we wish to find the maximum stress at each point and the stress is a function of the moment, thrust and shear and as has been pointed out it is not necessarily maximum at the same time that the moment is, but it is so nearly so that this is assumed to be the case. The maximum positive live load moment at point 0 is found by multiplying the sum of all of the positive quantities in column 67 by 437.5. The horizontal thrust which

TABLE 11.—*Computations for H_o, V_o, and M due to dead load*

64	65	66	67	68	69	70	71	72	73	74	75	76	77
Points	Unit load						D. L.	Dead load					
	H_o	V_o	M_o	M_2	M_{11}	M_o'		H_o	V_o	M_o	M_2	M_{11}	M_o'
1	0	1.0000	− 1.750	0	0	0	8,700	0	8,700	− 15,225	0	0	0
2	.048	.9980	− 4.300	+ .599	− .080	+ .570	6,660	320	6,647	− 28,638	+ 3,989	− 533	+ 3,796
3	.144	.9921	− 5.812	− 1.625	− .220	+ 1.665	5,015	722	4,975	− 29,147	− 8,149	− 1,103	+ 8,350
4	.285	.9796	− 6.210	− 3.091	− .387	+ 3.187	3,785	1,079	3,708	− 23,505	− 11,699	− 1,465	+ 12,063
5	.462	.9569	− 5.481	− 3.737	− .518	+ 4.942	2,980	1,377	2,852	− 16,333	− 11,136	− 1,544	+ 14,727
6	.657	.9213	− 3.801	− 3.629	− .530	+ 6.655	2,350	1,544	2,165	− 8,932	− 8,528	− 1,246	+ 15,639
7	.846	.8708	− 1.476	− 2.911	− .326	+ 8.000	1,900	1,607	1,655	− 2,804	− 5,531	− 619	+ 15,200
8	1.011	.8066	+ 1.142	− 1.801	+ .146	+ 8.799	1,530	1,547	1,234	+ 1,747	− 2,756	+ 223	+ 13,462
9	1.136	.7305	+ 3.717	− .514	+ .934	+ 8.922	1,245	1,414	909	+ 4,628	− 640	+ 1,163	+ 11,108
10	1.207	.6445	+ 5.943	+ .757	+ 2.085	+ 8.273	1,035	1,249	667	+ 6,151	+ 783	+ 2,158	+ 8,563
11	1.218	.5515	+ 7.619	+ 1.866	+ 3.624	+ 6.884	910	1,108	502	+ 6,933	+ 1,698	+ 3,298	+ 6,264
12	1.165	.4548	+ 8.572	+ 2.689	+ 2.079	+ 4.833	910	1,060	414	+ 7,800	+ 2,447	+ 1,892	+ 4,398
13	1.054	.3586	+ 8.758	+ 3.158	+ .941	+ 2.340	915	964	328	+ 8,014	+ 2,890	+ 861	+ 2,141
14	.892	.2671	+ 8.146	+ 3.215	+ .193	− .367	1,080	963	288	+ 8,798	+ 3,472	+ 208	− 396
15	.696	.1847	+ 6.862	+ 2.890	− .214	− 2.939	1,345	936	248	+ 9,229	+ 3,887	− 288	− 3,953
16	.488	.1153	+ 5.127	+ 2.267	− .357	− 4.992	1,730	844	199	+ 8,870	+ 3,922	− 618	− 8,636
17	.296	.0626	+ 3.281	+ 1.508	− .314	− 6.067	2,360	699	148	+ 7,743	+ 3,559	− 741	− 14,318
18	.145	.0276	+ 1.683	+ .798	− .191	− 5.860	3,245	471	90	+ 5,461	+ 2,590	− 620	− 19,016
19	.049	.0086	+ .587	+ .284	− .073	− 4.306	4,620	226	40	+ 2,712	+ 1,312	− 337	− 19,894
20	0	0	0	0	0	− 1.750	6,650	0	0	0	0	0	− 11,638
	11.799	10.9311	+ 61.437	+ 20.031	+ 10.002	+ 65.070	58,965	18,130	35,769	+ 78,086	+ 30,549	+ 9,803	+ 115,711
			− 28.830	− 17.308	− 3.210	− 26.281				−124,584	− 48,439	− 9,114	− 77,851
										− 46,498	− 17,890	+ 689	+ 37,860

TABLE 12.—*Computations for unit stresses*

78	79	80	81	82	83	84	85	86	87	88	89	90	91	92	93
										Extrados	Intrados	Maximum stresses			
		H	M	V	$H \cos \phi$	$V \sin \phi$	N	$\frac{N}{A}$	$M\frac{h}{2I}$	$\frac{N}{A}+M\frac{h}{2I}$	$\frac{N}{A}-M\frac{h}{2I}$	Extrados		Intrados	
												+	−	+	−
	D. L.	+ 18,130	− 46,498	+ 35,769	+ 9,953	+ 29,903	+ 39,856	+ 106	− 280	− 174	+ 386	− 174	− 174	+ 386	+ 386
Point 0	+C. L. L.	+	+		+		+	+	+						
sin ϕ=0.836.	+U. L. L.	+ 4,094	+ 26,879	+ 1,843	+ 2,248	+ 1,541	+ 3,789	+ 10	+ 162	+ 172	− 152	+ 172			− 152
cos ϕ=0.549.	−C. L. L.	+	−		+		+	+	−						
Area=377.	−U. L. L.	+ 1,068	− 12,613	+ 2,940	+ 586	+ 2,458	+ 3,044	+ 8	− 76	− 68	+ 84		− 68	+ 84	
$\frac{h}{2I}$=0.00603.	+T.	+ 761	+ 11,335	+ 55	+ 418	+ 46	+ 464	+ 1	+ 68	+ 69	− 67	+ 69			− 67
	−T.	− 1,014	− 15,113	− 73	− 557	− 61	− 618	− 2	− 91	− 93	+ 89		− 93	+ 89	
								Total excluding temperature stresses					− 242		
											Total	+ 67	− 335	+ 559	+ 167
	D. L.	+ 18,130	− 17,890	+ 27,069	+ 12,075	+ 20,193	+ 32,268	+ 110	− 180	− 70	+ 290	− 70	− 70	+ 290	+ 290
Point 2	+C. L. L.	+	+		+		+	+	+						
sin ϕ=0.746.	+U. L. L.	+ 3,175	+ 8,764	+ 1,607	+ 2,115	+ 1,199	+ 3,314	+ 11	+ 88	+ 99	− 77	+ 99			− 77
cos ϕ=0.666.	−C. L. L.	+	−		+		+	+	−						
Area=293.	−U. L. L.	+ 1,987	− 7,572	+ 2,738	+ 1,323	+ 2,043	+ 3,366	+ 12	− 76	− 64	+ 88		− 64	+ 88	
$\frac{h}{2I}$=0.01005.	+T.	+ 761	+ 6,221	+ 55	+ 507	+ 41	+ 548	+ 2	+ 62	+ 64	− 60	+ 64			− 60
	−T.	− 1,014	− 8,297	− 73	− 675	− 54	− 729	− 3	− 83	− 86	+ 80		− 86	+ 80	
								Total excluding temperature stresses					− 134		
											Total	+ 93	− 220	+ 458	+ 153
	D. L.	+ 18,130	+ 689	+ 569	+ 18,112	+ 26	+ 18,138	+ 113	+ 25	+ 138	+ 88	+ 138	+138	+ 88	+ 88
Point 11	+C. L. L.	+	+		+		+	+	+						
sin ϕ=0.046.	+U. L. L.	+ 3,361	+ 4,376	+ 356	+ 3,358	+ 16	+ 3,374	+ 21	+ 158	+ 179	− 137	+ 179			− 137
cos ϕ=0.999.	−C. L. L.	+	−		+		+	+	−						
Area=161.	−U. L. L.	+ 1,801	− 1,404	+ 51	+ 1,799	+ 2	+ 1,801	+ 11	− 51	− 40	+ 62		− 40	+ 62	
$\frac{h}{2I}$=0.0361.	+T.	+ 761	− 1,803	+ 55	+ 760	+ 3	+ 763	+ 5	− 65	− 60	+ 70		− 60	+ 70	
	−T.	− 1,014	+ 2,403	− 73	− 1,013	− 3	− 1,016	− 6	+ 87	+ 81	− 93	+ 81			− 93
								Total excluding temperature stresses							− 49
											Total	+ 398	+ 38	+ 220	− 142
	D. L.	+ 18,130	+ 37,860	− 23,196	+ 10,117	+ 19,253	+ 29,370	+ 78	+ 228	+ 306	− 150	+ 306	+ 306	− 150	− 150
Point 0′	+C. L. L.	+	+		+		+	+	+						
sin ϕ−0.830.	+U. L. L.	+ 4,039	+ 28,468	− 1,197	+ 2,254	+ 994	+ 3,248	+ 9	+ 172	+ 181	− 163	+ 181			− 163
cos ϕ=0.558.	−C. L. L.	+	−		+		+	+	−						
Area=377.	−U. L. L.	+ 1,123	− 11,498	− 2,771	+ 627	+ 2,300	+ 2,927	+ 8	− 69	− 61	+ 77		− 61	+ 77	
$\frac{h}{2I}$=0.00603.	+T.	+ 761	+ 11,380	+ 55	+ 425	− 46	+ 379	+ 1	+ 69	+ 70	−68	+ 70			− 68
	−T.	− 1,014	− 15,153	− 73	− 566	+ 61	− 505	− 1	− 91	− 92	+90		− 92	+ 90	
								Total excluding temperature stresses						− 73	− 313
											Total	+ 557	+ 153	+ 17	− 381

$$N = H \cos \phi + V \sin \phi \qquad f_c = \frac{N}{A} \pm M \frac{h}{2I}$$

occurs at the same time as this maximum moment is found by adding up the quantities in column 65 for the points which give a positive moment and multiplying the sum by 437.5 The vertical shear which occurs at the same time as the maximum moment is found in the same way. The maximum negative moment and the thrust and shear which occur at the same time are found by placing loads at all of the points which give negative moments. These values are recorded in the proper places in Table 12.

The live-load moments and thrusts at the other points are found in the same way. In determining the shear for the other points it should be remembered that V_o in column 66 is the coefficient for the left reaction which is equal to the coefficient for the vertical shear at point 0, but not at other points if there are loads between the point where the shear is desired and the left reaction. If there are any loads to the left of the point under consideration, the shear is found by adding up the proper quantities in column 66 and then

subtracting 1.00 for each load to the left. The quantity thus obtained is multiplied by the live load per load point, as shown in the following computations:

Live-load moments, thrusts, and shears

Live load = 125 pounds per square foot.
= 125×3.5 = 437.5 pounds per load point

Point 0:	
$+M=437.5\times 61.437=$	26,879
$H=437.5\times 9.357=$	4,094
$V=437.5\times 4.212=$	1,843
$-M=437.5\times 28.830=$	12,613
$H=437.5\times 2.442=$	1,068
$V=437.5\times 6.719=$	2,940
Point 2:	
$+M=437.5\times 20.031=$	8,764
$H=437.5\times 7.258=$	3,175
$V=437.5\times(4.673-1.00)=$	1,607
$-M=437.5\times 17.308=$	7,572
$H=437.5\times 4.541=$	1,987
$V=437.5\times 6.258=$	2,738
Point 11:	
$+M=437.5\times 10.002=$	4,376
$H=437.5\times 7.683=$	3,361
$V=437.5\times(3.813-3.0)=$	356
$-M=437.5\times 3.210=$	1,404
$H=437.5\times 4.116=$	1,801
$V=437.5\times(7.117-7.0)=$	51
Point 0′:	
$+M=437.5\times 65.070=$	28,468
$H=437.5\times 9.233=$	4,039
$V=437.5\times(10.265-13.0)=$	1,197
$-M=437.5\times 26.281=$	11,498
$H=437.5\times 2.566=$	1,123
$V=437.5\times(0.666-7.0)=$	2,771

These moments, thrusts, and shears are tabulated in the proper places in columns 80, 81, and 82.

The values of V_t are found from the equation

$$V_t=\frac{\frac{2r}{(dx)^2}+20\frac{G}{C}}{F-B\frac{G}{C}}etE$$

In substituting in this equation, e, the coefficient of expansion of the concrete, is assumed to be 0.000006. E, the modulus of elasticity of the concrete, is taken as 2,000,000 pounds per square inch or 288,000,000 pounds per square foot, and t has been assumed as $+30°$ F. and $-40°$ F. These values with other values already determined when substituted in the equation give values of V_t as $+55$ and -73.

The formula for H_t given on page 95 reduces to

$$H_t=\frac{V_tB+34{,}560t}{C}=\begin{matrix}+761\\-1{,}014\end{matrix}$$

The values of V_t and H_t are the same for every point.

M_t is different for every point and substitution in the formula on page 95 gives the following:

For point 0,

$$M_{t,o}=-V_t\frac{dx\Sigma z\Delta}{2\ \Sigma\Delta}+H_t\frac{\Sigma y\Delta}{\Sigma\Delta}=\begin{matrix}+11{,}335\\-15{,}113\end{matrix}$$

For point 2,

$$M_{t,2}=M_{t,o}+3V_t\frac{dx}{2}-7.1H_t=\begin{matrix}+6{,}221\\-8{,}297\end{matrix}$$

For point 11,

$$M_{t,11}=M_{t,o}+21V_t\frac{dx}{2}-19.92H_t=\begin{matrix}-1{,}803\\+2{,}403\end{matrix}$$

For point 0′,

$$M_{t,o}'=M_{t,o}+40V_t\frac{dx}{2}-5H_t=\begin{matrix}+11{,}380\\-15{,}153\end{matrix}$$

Table 12 is completed in the same manner as the similar table for the symmetrical arch.

Stresses in steel and concrete.—As in the example for the symmetrical arch, the maximum stresses in Table 12 should be examined to determine if the tensile strength of the concrete will be exceeded. In the present example the tension in the concrete at the extrados at point 0 is 335 pounds per square inch, the concrete will crack and the tension must be taken by the steel. The stresses should therefore be computed in accordance with the theory of flexure and direct stress. The calculations for point 0 differ from those given in the example for a symmetrical arch as there is reinforcement in both the top and bottom of the arch ring. In this case the diagrams on page 399 to 402 of Hool and Johnson's Concrete Engineers' Handbook are used and it will be assumed that the reader is familiar with these diagrams and the theory related to them. It should be noted that x_o and t in Hool and Johnson's diagrams are the same as u and h in this article.

The following calculations are made to determine the stress in steel and concrete at point 0. The worst condition at point 0 is caused by a negative moment due to dead load, maximum negative moment due to live load, and a negative moment due to a fall of temperature.

	M_o	N
D. L.	−46,498	+39,856
−L. L.	−12,613	+3,044
−T.	−15,113	−618
	−74,224	+42,282

$u=\frac{-74224}{+42282}=-1.76$ feet, eccentricity of thrust.

$\frac{u}{h}=\frac{1.76}{2.5}=0.70$ feet (value of $\frac{x_o}{t}$ in Hool and Johnson).

$\frac{d'}{h}=\frac{.17}{2.5}=0.068$ (d' = distance from center of steel to surface of concrete).

$P_o=\frac{2\times .5625}{144\times 2.5}=.0031$ per cent steel.

From the diagram on page 399 of Hool and Johnson (for $d'=0.05t$ in which t is the same as h) with $P_o=0.0031$ and $\frac{u}{h}=0.70$, k is equal to 0.310, and from diagram on page 400 (for $d'=0.10t$), k is equal to 0.300. As $\frac{d'}{h}$ s 0.068 we should use a value of k between the two or bout 0.305.

The diagram on page 402 is for $\frac{d'}{h}=0.10$.

To use the diagram for $\frac{d'}{h}=0.05$, P_o should be divided by 0.790 according to instructions below the diagram. As the value of $\frac{d'}{h}$ in this case is 0.068 we should divide P_o by about 0.84, which gives a value of $P_o=0.0037$ for use with the diagram. Using a value of $P_o=0.0037$ and $k=0.305$ we find from the diagram that $L=0.0900$. Then substituting in the equation given by Hool and Johnson, $f_c=\frac{M}{Lbt^2}=\frac{12\times 74{,}224}{0.0900\times 12\times(30)^2}=$ 916 pounds per square inch, and $f_s=nf_c\left(\frac{d}{kt}-1\right)=15\times 916\left(\frac{28}{305\times 30}-1\right)=28{,}300$ pounds per square inch.

Stresses are computed at all points where Table 12 indicates that the concrete will crack and if necessary the design of the arch ring is revised. Usually a slight modification is sufficient to keep within the limits of the specifications.

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